Plasma Science and Technology

PAPER

The instability of terahertz plasma waves in cylindrical FET

To cite this article: Dongao LI *et al* 2019 *Plasma Sci. Technol.* **21** 045002

 Plasma Science and Technology

Plasma Sci. Technol. **21** (2019) 045002 (4pp) https://doi.org/10.1088/2058-6272/aaf874

The instability of terahertz plasma waves in cylindrical FET

Dongao LI (李东澳), Liping ZHANG (张丽萍) and Hongmei DU (杜洪梅)

School of Sciences, Lanzhou University of Technology, Lanzhou 730050, People's Republic of China

E-mail: zhanglp@lut.cn

Received 27 September 2018, revised 10 December 2018
Accepted for publication 13 December 2018
Published 31 January 2019

Abstract

In this paper, the Dyakonov–Shur instability of terahertz (THz) plasma waves has been analyzed in gated cylindrical field effect transistor (FET). In the cylindrical FET, the hydrodynamic equations in cylindrical coordinates are used to describe the THz plasma wave in two-dimensional electronic gas. The research results show that the oscillation frequency of the THz plasma wave is increased by increasing the component of wave in the circumferential direction, but instability increment of the THz plasma wave are increased by increasing the radius of channel.

Keywords: THz plasma waves, cylindrical field effect transistor, oscillation frequency

(Some figures may appear in colour only in the online journal)

1. Introduction

More and more people have paid attention to terahertz (THz) waves due to the potential application in medical, communications, micro-nano optics, etc [1–3]. There are many ways to generate THz waves. Based on the extensive research of electromagnetic properties, both the electronic and the optical methods can produce THz radiation [4–6]. In 1990s, Dyakonov and Shur [7] noticed that the plasma wave can become unstable in the channel of field effect transistor (FET) with asymmetric boundary conditions. Under the asymmetric boundary condition, the channel of FET can act as a resonator to amplify the plasma wave, when the plasma wave reflects at the boundaries, which generate the electromagnetic radiation with THz frequency.

Subsequently, various rectangular models with the Dyakonov and Shur instability have been reported. Some scientists consider the effect of electrons. For two-dimensional electron gas (2DEG) in channel of FET, the mean free path of electron–electron collisions is less than both the channel length and the mean free path of collisions with impurities and/or phonons, which causes the electron scattering and the viscosity of plasma [8, 9]. Most scientists have concerned about the influence of geometry of device on THz plasma waves [10–13]. Dyakonov and Shur used a new equation to replace the gradual-channel equation and noted that the instability in ungated FETs was similar to gated FETs with asymmetrical boundary conditions [12]. But in the actual rectangular FET, the width of the gate is much larger than the length, $\frac{W}{L} \sim 100$, which means that the THz plasma wave must have two wave vectors in the x direction and the y direction, respectively. Dyakonov analyzed the oblique propagation of THz plasma waves with the two-dimensional hydrodynamic equations in rectangular coordinates [13].

In recent reports, some scientists are interested in the plasma oscillation in other geometric devices (non-rectangular FET). Sydoruk *et al* analyzed the instability of THz plasmas waves in gated FET of Corbino geometry with symmetric boundary conditions and found that the lowest eigenfrequency is twice as high as rectangular FET [14]. Rahmatallahpur *et al* analyzed the plasma oscillation in the gated [15] and the ungated [16] cylindrical FET with the hydrodynamic equation and Poisson's equation. For the gated cylindrical FET, they replaced the effect of gate on channel with the effect of a new charge density [15]. In this case, the gated problem can be easily solved by the way of ungated problem. This model can avoid the absorption of radiation power in metallic gate, and improve the efficiency of radiation power, and the analytical expression of the total radiation power with the various transistor parameters has been given. In [16], Rahmatallahpur *et al* noticed that the plasma frequency changes logarithmically with the radius of channel, when the radius is smaller, and the variation trend of plasma frequency is the same as that in flat 2DEG at the large radius.

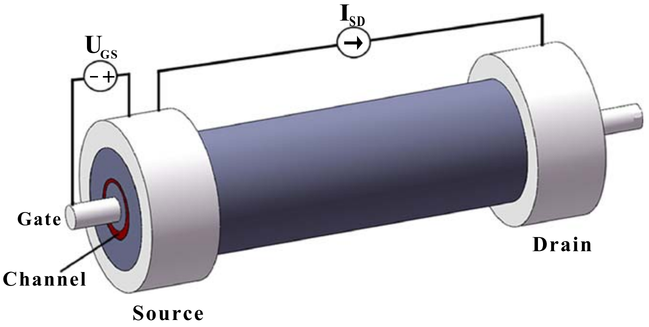

Figure 1. Coaxially center-gated field effect transistor with appropriate biasing.

In this paper, we investigated the Dyakonov–Shur instability of THz plasma waves by using the hydrodynamic equations in gated cylindrical FET (see figure 1). The cylindrical FET is a resonator in axis direction and a waveguide in circumferential direction. When the wave number of circumferential direction is not zero, the oscillation mode of THz plasma waves in gated cylindrical FET is different from that in the one-dimensional rectangular FET. The plasma wave propagation in the cylindrical FET is spiral between the source and drain. Both the wave number of circumferential direction and the radius of channel can influence the oscillation frequency and the instability increment.

2. Model

When the density of electrons is very high, the electron–electron scattering rate is much higher than other frequency scales. In this case, the action of electrons in channel is similar to that of fluids, and can be described by hydrodynamic models. To describe the behavior of 2DEG in gated cylindrical FET, the hydrodynamic equations (continuity equation and motion equation) in cylindrical coordinates are used:

$$\frac{\partial n}{\partial t} + \frac{1}{r}\frac{\partial n \boldsymbol{v}_{c}}{\partial \theta} + \frac{\partial n \boldsymbol{v}_{z}}{\partial z} = 0, \tag{1}$$

$$\frac{\partial \boldsymbol{v}_{c}}{\partial t} + \boldsymbol{v}_{c}\frac{1}{r}\frac{\partial \boldsymbol{v}_{c}}{\partial \theta} + \boldsymbol{v}_{z}\frac{\partial \boldsymbol{v}_{c}}{\partial z} = -\frac{e}{m_{e}}\frac{1}{r}\frac{\partial \varphi}{\partial \theta}, \tag{2}$$

$$\frac{\partial \boldsymbol{v}_{z}}{\partial t} + \boldsymbol{v}_{c}\frac{1}{r}\frac{\partial \boldsymbol{v}_{z}}{\partial \theta} + \boldsymbol{v}_{z}\frac{\partial \boldsymbol{v}_{z}}{\partial z} = -\frac{e}{m_{e}}\frac{\partial \varphi}{\partial z}, \tag{3}$$

where n is the electron density, r is the radius of channel, $\boldsymbol{v}_c$ and $\boldsymbol{v}_z$ are the velocity in circumferential direction (θ direction) and axis direction (z direction) respectively, e is the electron charge, m_e is the electron effective mass and φ is the potential. The φ is equal to $U_{GC} - U_T$, where U_{GC} is the gate-to-channel voltage and U_T is the threshold voltage.

The gradual-channel equation is as follow [7]:

$$en = C\varphi, \tag{4}$$

where C is the capacitance per unit of the gated area. When the distance between the gate and the channel is small enough, the equation (4) is valid.

3. Plasma waves instability

As is usually done for the analysis of instabilities, we let $n = n_0 + n_1$, $\boldsymbol{v}_c = \boldsymbol{v}_{c1}$, $\boldsymbol{v}_z = \boldsymbol{v}_0 + \boldsymbol{v}_{z1}$, $\varphi = \varphi_0 + \varphi_1$ with n_0, $\boldsymbol{v}_0$, φ_0 are constant values and n_1, $\boldsymbol{v}_{c1}$, $\boldsymbol{v}_{z1}$, φ_1 are small perturbations. Using n_1, $\boldsymbol{v}_{c1}$, $\boldsymbol{v}_{z1}$, $\varphi_1 \sim \exp(-\mathrm{i}\omega t + \mathrm{i}m\theta + \mathrm{i}kz)$ to linearize equations (1)–(3), where $\frac{\partial}{\partial t} = -\mathrm{i}\omega$, $\frac{1}{r}\frac{\partial}{\partial \theta} = \mathrm{i}\frac{m}{r}$, $\frac{\partial}{\partial z} = \mathrm{i}k$.

Then we have:

$$(\omega - \boldsymbol{v}_0 k)n_1 - n_0\left(\frac{m}{r}\boldsymbol{v}_{c1} + k\boldsymbol{v}_{z1}\right) = 0, \tag{5}$$

$$(\omega - \boldsymbol{v}_0 k)\boldsymbol{v}_{c1} = \frac{e}{m_e}\frac{m}{r}\varphi_1, \tag{6}$$

$$(\omega - \boldsymbol{v}_0 k)\boldsymbol{v}_{z1} = \frac{e}{m_e}k\varphi_1, \tag{7}$$

where m is the wave number of θ direction and k is the wave number of z direction. Due to the periodic boundary condition along θ direction of the tube, we have $n_1(\theta + 2\pi) = n_1(\theta)$, so that m is integer ($m = 0, 1, 2, \cdots$) [15, 16].

From equations (4)–(7), we can find the dispersion equation of the THz plasma wave with the form

$$(\omega - \boldsymbol{v}_0 k)^2 = s^2\left(\frac{m^2}{r^2} + k^2\right), \tag{8}$$

where $s = \sqrt{\frac{\varphi_0 e}{m_e}}$ is plasma wave velocity.

From equation (8), two roots can be found for k_1 and k_2 with the form

$$k_{1,2} = \frac{-\omega \boldsymbol{v}_0 \pm \sqrt{\omega^2 - (s^2 - \boldsymbol{v}_0^2)\frac{m^2}{r^2}}}{s^2 - \boldsymbol{v}_0^2}. \tag{9}$$

When $m = 0$, $k_{1,2} = \frac{\omega}{\boldsymbol{v}_0 \pm s}$, waves propagate only along the direction of z. In this case, it is same to the one-dimensional rectangular FET [7].

Introducing the length of tube (L), to rewrite the dispersion equation (equation (8)) and get a dimensionless equation for dispersion relation. The dimensionless dispersion equation has the form

$$(\Omega - \beta K)^2 = \left(\frac{M^2}{R^2} + K^2\right), \tag{10}$$

where $\Omega = \frac{\omega}{s}L$, $\beta = \frac{\boldsymbol{v}_0}{s}$, $R = \frac{r}{L}$, $K = kL$, $M = m = 0, 1, 2, \cdots$, are the dimensionless frequency, the Mach number, the dimensionless radius, the dimensionless wave vector of z direction and the dimensionless wave vector of θ direction, respectively.

The dyakonov and shur's asymmetrical boundary conditions are used in the cylindrical fet, the zero ac potential (φ_1) at the source and the zero ac conduction current ($j = env$) at the drain. The $n_1 v_1$ is ignored because it is the high order small quantity. The boundary conditions are as follows:

$$n_1(z = 0) = 0, \tag{11}$$

$$J(z = L) = \varphi_0 \boldsymbol{v}_{z1} + \boldsymbol{v}_0 \varphi_1 = 0. \tag{12}$$

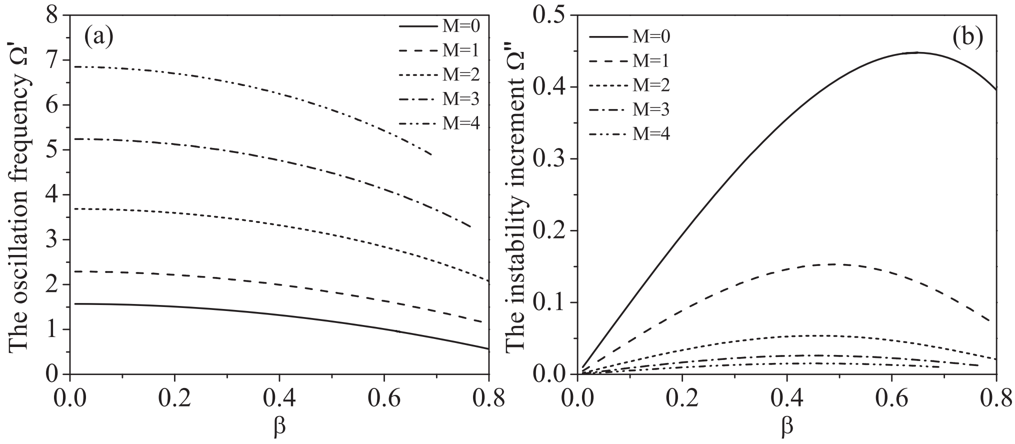

Figure 2. The oscillation frequency Ω' and instability increment Ω'' as a function of β for different M with $R = 0.6$.

The $\boldsymbol{v}_{z1}$ comes from equation (7) with $\varphi_1 = A\,\exp(\mathrm{i}k_1L) + B\,\exp(\mathrm{i}k_2L)$, where A and B are constants, and the $\boldsymbol{v}_{z1}$ has the form

$$\boldsymbol{v}_{z1} = \frac{\frac{e}{m_e}k_1}{\omega - \boldsymbol{v}_0k_1}A\,\exp(\mathrm{i}k_1L) + \frac{\frac{e}{m_e}k_2}{\omega - \boldsymbol{v}_0k_2}B\,\exp(\mathrm{i}k_2L). \quad (13)$$

Dimensionless processing of equations (11)–(13) (same as equation (10)), we can obtain

$$\exp[\mathrm{i}(K_1 - K_2)] = \frac{\frac{K_2}{\Omega - \beta K_2} + \beta}{\frac{K_1}{\Omega - \beta K_1} + \beta}, \quad (14)$$

where the K_1 and K_2 can be found from equation (10).

Equation (14) is a transcendental equation that can be only solved numerically. We can find both the real and imaginary parts of the complex frequency $\Omega = \Omega' + \mathrm{i}\Omega''$ by separating real and imaginary parts. When the plasma oscillation growth rate becomes positive, that is, $\Omega'' > 0$, the THz plasma wave is called unstable.

From equations (10) and (14), the oscillation frequency (Ω') and the instability increment (Ω'') depend on three parameters: the electron drift velocity, the θ direction wave number and the radius of channel. Figure 2 shows the dependence of the oscillation frequency and the instability increment on β for different wave numbers of θ direction with $R = 0.6$. The value of M can change the mode of oscillation of the THz plasma wave. When $M = 0$, the THz plasma wave propagates along the direction of z, otherwise the wave propagates spirally along axis. From the equation (9), we can find the wave numbers of z direction (k_1 and k_2) are related to the wave numbers of the θ direction (m). The oscillation frequency and the instability increment can be changed by the numerical size of the wave numbers of z direction. As shown in figure 2(a), the oscillation frequency increases with the increase of wave numbers of θ direction, however, the oscillation frequency decreases with the increase of electron drift velocity. Therefore, in order to improve the frequency of THz plasma waves in cylindrical FET, the wave should be closer to the circumferential direction during propagation. Figure 2(b) shows that the instability increment decreases with the increase of M. The peak of instability increment moves to the lower β when the M is increased, and the instability increment approaches zero when $M > 4$. This shows that the closer the direction of propagation of plasma waves is to the direction of axis in cylindrical FET, the more unstable it becomes.

In the case of GaAs FET, the length of tube (L) is 200 nm and the radius of channel is 120 nm. The increment $f = \frac{\omega''}{2\pi} = \frac{\Omega'' s}{2\pi L}$, where Ω'' is the dimensionless instability increment with $R = 0.6$, exceeds the decrement $1/\tau_{\mathrm{p}}$ when $M = 0,\ 1$ and $0.12 < \beta < 0.97,\ 0.26 < \beta < 0.70$, respectively (assuming momentum relaxation time of electron $\tau_{\mathrm{p}} \approx 10^{-11}$ s at 77 K). With the increase of the wave number of θ direction, the plasma wave propagation length is increasing, which enhances the frequency of electron–electron collisions and loss of THz plasma.

Figure 3 plots the dependance of the oscillation frequency and the instability increment on β for different radii of channel with $M = 1$. When $M = 0$, $\frac{M}{R} = 0$, the oscillation frequency and the instability increment cannot be influenced by the radius of channel. It can be seen from figure 3, when the radius decreases, the oscillation frequency increases but the instability increment decreases. In the case of $M = 1$, the surface area of channel is increased by increasing the radius of channel, which results in a decrease of wave numbers of z direction. That means when the wavelength increases, the oscillation frequency decreases.

In a sample of GaAs FET, the length of tube (L) is 200 nm and the wave number of θ direction is $m = 1$, The increment $f = \frac{\omega''}{2\pi} = \frac{\Omega'' s}{2\pi L}$, where Ω'' is the dimensionless instability increment with $M = 1.0$, exceeds the decrement $1/\tau_{\mathrm{p}}$ when $r = 120,\ 160,\ 200$ nm and $0.26 < \beta < 0.70$, $0.19 < \beta < 0.79$, $0.16 < \beta < 0.83$, respectively (assuming $\tau_{\mathrm{p}} \approx 10^{-11}$ s at 77 K). By increasing the radius of channel, the surface area of channel is increased, which enhances both the number (electron density is 10^{12} cm^{-2} in semiconductor FET) and the collision frequency of electron.

And then, the dependence of the oscillation frequency on R with $\beta = 0.2$ is plotted in figure 4. In figure 4, the slope of the curve is greater at lower R, that is, the oscillation frequency decreases quickly for a small radius of channel. However, the oscillation frequency decreases slowly for higher radius of channel.

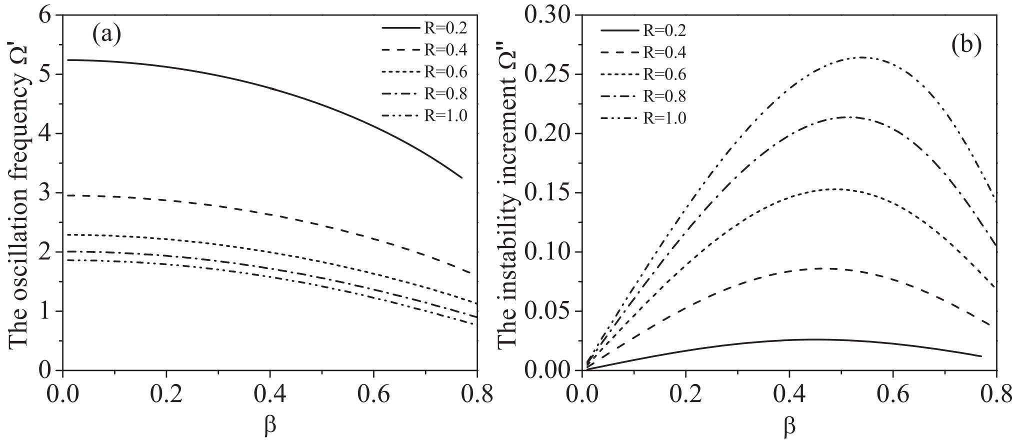

Figure 3. The oscillation frequency Ω' and instability increment Ω'' as a function of β for different R with $M = 1.0$.

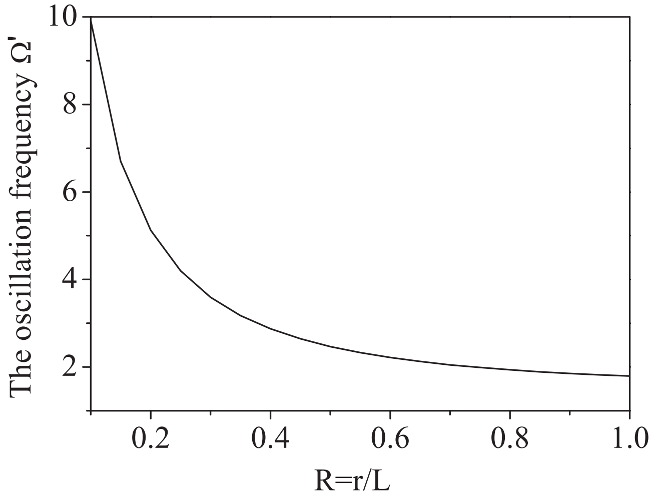

Figure 4. The oscillation frequency Ω' as a function of R with $\beta = 0.2$ and $M = 1.0$.

In the gated cylindrical FET with the Dyakonov–Shur instability, the oscillation frequency reaches the THz frequency. For example, assuming that the length of tube (L) is 200 nm, the radius of channel (r) is 120 nm, the wave number of θ direction (m) is 1, the electronic velocity (v_0) is 10^7 cm s^{-1}, the plasma wave velocity (s) is 10^8 cm s^{-1}. That means the dimensionless parameters are $R = 0.6$, $M = 1$, $\beta = 0.1$, which can let us find the dimensionless frequency Ω' is 2.27. According to the above parameters, the true oscillation frequency $f = \frac{\omega'}{2\pi} = \frac{\Omega' s}{2\pi L} \approx 1.8$ THz.

4. Conclusions

In the gated cylindrical FET, the hydrodynamic equations in cylindrical coordinates are used to describe the THz plasma wave in two-dimensional electronic gas. Due to the reflection of the boundary, the current becomes unstable and the plasma wave with THz frequency is amplified. We found that the THz plasma wave in the device with differen structures has different dispersion relationship and oscillation modes. The oscillation frequency and the instability increment can be influenced by the wave number of circumferential direction (θ direction) and the radius of channel. Numerical results show that the frequency of the THz plasma wave is increased by increasing the component of wave in the circumferential direction, but instability increment of the THz plasma wave is increased by increasing the radius of channel. This paper is also helpful for experimental observation of the THz plasma wave.

Acknowledgments

This work was supported by National Natural Foundation of China (No. 10975114), Research Projects of Higher Education of Gansu Province (No. 2017A-016) and HongLiu first-class disciplines Development Program of Lanzhou University of Technology.

References

[1] Lin L *et al* 2018 *Mater. Res. Express* **5** 045802
[2] Zeitler J A and Gladden L F 2009 *Eur. J. Pharm. Biopharm.* **71** 2
[3] Wang X X *et al* 2018 *AIP Adv.* **8** 105029
[4] Siles J V and Grajal J 2010 *IEEE Trans. Microw. Theory Tech.* **58** 1933
[5] Ding Y J and Shi W 2006 *Solid-State Electron.* **50** 1128
[6] Tonouchi M 2007 *Nat. Photon.* **1** 97
[7] Dyakonov M and Shur M 1993 *Phys. Rev. Lett.* **71** 2465
[8] Cheremisin M V and Samsonidze G G 2008 *Solid-State Electron.* **52** 338
[9] Dmitriev A P, Furman A S and Kachorovskii V Y 1996 *Phys. Rev.* B **54** 14020
[10] Zhang L P 2016 *Plasma Sci. Technol.* **18** 360
[11] Wang D P *et al* 2018 *Frequenz* **72** 471
[12] Dyakonov M and Shur M 2005 *Appl. Phys. Lett.* **87** 111501
[13] Dyakonov M I 2008 *Semiconductors* **42** 984
[14] Sydoruk O, Syms R R A and Solymar L 2010 *Appl. Phys. Lett.* **97** 263504
[15] Rahmatallahpur S and Rostami A 2016 *Superlattices Microstruct.* **97** 176
[16] Rahmatallahpur S and Rostami A 2016 *Optik* **127** 8294