Plasma Science and Technology

PAPER

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Plasma Sci. Technol. **21** (2019) 075301 (8pp) https://doi.org/10.1088/2058-6272/ab0bcd

Optimized analysis of ionospheric amplitude modulated heating parameters for excitation of very/extremely low frequency radiations

Jutao YANG (杨巨涛)[1,2], Jianguo WANG (王建国)[1,3], Qingliang LI (李清亮)[2], Haiqin CHE (车海琴)[2] and Shuji HAO (郝书吉)[2]

[1] School of Electronic and Information Engineering, Xi'an Jiaotong University, Xi'an 710049, People's Republic of China
[2] National Key Laboratory of Electromagnetic Environment, China Research Institute of Radiowave Propagation, Qingdao 266107, People's Republic of China
[3] Northwest Institute of Nuclear Technology, Xi'an 710024, People's Republic of China

E-mail: liql220103@126.com

Received 18 September 2018, revised 24 February 2019
Accepted for publication 1 March 2019
Published 17 April 2019

Abstract

It is now well known that amplitude modulated (AM) high frequency (HF) radio wave transmissions into the ionosphere can be used to generate very/extremely low frequency (VLF/ELF) radio waves using the so-called 'electrojet antenna'. Duty cycle and heating frequency are analyzed and discussed with the lower-ionosphere modulated heating model, so as to improve the radiation efficiency of VLF/ELF waves in AM ionospheric heating experiments. Based on numerical simulation, the ranges of parametric selectivity in optimal duty cycle and heating frequency (f_{HF}) are derived. The International Reference Ionosphere 2015 (IRI-2015) model and two-parameter model are used to predict background electron density profiles, and optimized ranges of duty cycle for different density profiles are analyzed and compared. The influences of wave polarizations on optimal duty cycle are also discussed. It is shown that intensity of the VLF/ELF equivalent radiation source (**M**) firstly rises and then falls with the increase of duty cycle. When using the IRI model, **M** peaks at a duty cycle of 50%, optimally ranging from 40%−70%. For the two-parameter model case, an optimal duty cycle is 40% and the optimized ranges vary from 30%−60%. Heating with an X-mode polarization is more efficient than with the O-mode case in VLF/ELF wave generation. Nevertheless, an optimal duty cycle is almost independent of HF wave polarizations. To obtain better VLF/ELF generation, optional f_{HF} may be 0.8−0.9 times of f_oE for the O-mode heating and 0.75−0.85 times for the X-mode polarization case. Finally, the variations of these two parameters in different latitudes are discussed.

Keywords: amplitude modulated heating, parameter optimization, duty cycle, heating frequency

(Some figures may appear in colour only in the online journal)

1. Introduction

As the main frequency band used in submarine communication and navigation, very/extremely low frequency (VLF/ELF) has extremely long wavelength. Thus huge ground transmitting stations are necessary for building up such communication and navigation systems, with coverage of several square kilometers, which are excessively large investments, easily damaged in wartime, and hard to repair once destroyed. In the 1970s, Willis and Davis [1] suggested the theory of artificial heating of the ionosphere with modulated high frequency (HF) waves for the first time. They supposed that modulated oscillations of the electrojet in the ionospheric E layer could be generated and the expected ELF wave radiation could be excited by transmitting HF pulse waves into the ionosphere. Soon afterwards in 1974, Getmantsev *et al* [2] performed modulated HF heating experiments near Gor'kii in the USSR and detected VLF/ELF signals around 1.2–7 kHz at 180 km north of the heater. Later, Stubbe *et al* [3] repeated experimentally generation of VLF/ELF radiation with the above mentioned method at Tromsø in Norway and Barr *et al* [4] successfully received VLF/ELF waves 2050 km away from the HF heating facility at Tromsø. Moreover, successively experimental and theoretical investigations [5–7] have validated the feasibility of applications of VLF/ELF signals excited by modulated heating of the ionosphere in submarine communication and navigation.

Recently, with further improvement of the experimental ability, especially after the upgrade of the heater at High-frequency Active Auroral Research Program (HAARP) with an effective radiation power (ERP) of 3.6 GW [8], the radiation intensity and propagation distance of excited VLF/ELF signals in ionospheric heating experiments have been greatly enhanced. Moore *et al* [9] presented VLF/ELF measurements generated by modulated HF heating of the auroral electrojet by HAARP (power of the transmitter is 960 kW) at a ground distance more than 4400 km with strength levels 15–20 fT, and Cohen *et al* [10] used the upgraded HAARP to conduct experiments and received 60 fT VLF/ELF signals at the same location. Nevertheless, due to the relatively small signal intensity of radiation waves and rather low conversion efficiency of HF into VLF/ELF of about 0.001%, the practical use of VLF/ELF waves generated by modulated HF heating as radiation sources is limited. Thus investigations on how to improve radiation efficiency of VLF/ELF waves have become a popular direction in the present relevant research, the basis of which is to study influencing factors of the excited VLF/ELF radiation intensity [11–16].

There are two major approaches to increasing the radiation efficiency of VLF/ELF generation by modulated HF heating of the ionosphere with consideration of influencing factors. The first one is to improve the available modulated heating methods. For example, Papadopoulos *et al* [17] proposed a method of fast sweeping over the heated region to enlarge perturbed areas so as to enhance the radiation intensity of the resulting VLF/ELF waves, which may have an efficiency of about two orders of magnitude higher than the usual way. Afterwards, Cohen *et al* [10, 18] performed fast sweeping heating experiments at HAARP and found the resultant radiation intensity cannot be improved by such high order of magnitude but still has a remarkable enhancement in amplitude by 4–6 dB. Moreover, Cohen *et al* [10, 18] also pointed out geometric modulation enables a much larger area in the ionosphere to be heated with 7–11 dB enhanced amplitudes for receivers at long range (>500 km). Geometric modulation, however, sometimes decreased the signal strength at short range (<100 km). Subsequently, Fujimaru [19] optimized the heating pattern used to generate the ionospheric source region, to achieve a similar 10 dB efficiency enhancement specifically for the short range signal. Considering effects of the background electron density, Milikh *et al* [20] proposed a method of preheating the ionosphere followed by normal amplitude modulated (AM) heating at VLF frequencies to enhance the VLF/ELF generation efficiency. By preheating, the ambient electron density and the density gradient can be enhanced, which leads to an increase in VLF radiation intensity. It is shown that following the long preheating pulse by AM heating at VLF frequencies can result in an elevation of VLF/ELF radiation intensity up to 7 dB over the non-preheated case. Cohen *et al* [21] provided direct experimental evidence that the amplitude of VLF/ELF waves may be strengthened by a preheating effect. Moore *et al* [22] reported VLF/ELF wave generation by powerful HF waves using the thermal cubic nonlinearity of the ionosphere, which is independent of the modulation of natural currents. The second way to promote the efficiency of VLF/ELF radiation is to optimize the heating parameters. Papadopoulos *et al* [17] presented that VLF/ELF generation efficiency improved with the increase of the ERP of the HF pumping wave. Kuo *et al* [23] demonstrated that the half wave rectification is better than others by comparison of several typical AM waveforms used in HF heating experiments. Stubbe *et al* [3] considered that the intensity of VLF/ELF signals can be elevated by about 3 dB in the case of heating at X-mode polarization compared with the normal O-mode. Milikh *et al* [24] experimentally demonstrated that the excited magnetic intensity (B) of VLF/ELF waves is inversely proportional to the heating frequency (f_{HF}) in the form of $B \sim f_{\mathrm{HF}}^{-\beta}$ with $\beta = 1$–2. Moreover, Hao *et al* [25] concluded that heating effects are closely related to heating parameters as well as background ionosphere such as the critical frequency. They also pointed out that since the experimental heating frequency used by Milikh *et al* [24] is much lower than the critical frequency of the lower ionosphere, the above relationship of B with f_{HF} may not be universally applicable. Besides, it has been demonstrated that duty cycle has a significant influence on the radiation efficiency of VLF/ELF generation in theoretical and experimental investigations at high latitudes [4, 7, 10], which has not yet been mentioned in mid-low latitude ionosphere.

We investigate heating parameters by using the ionospheric modulated heating model, then make a thorough investigation of the influences of duty cycle and heating frequency on VLF/ELF radiation efficiency in modulated HF heating of the ionosphere at mid-low latitudes, and finally obtain optimal ranges of the above-mentioned two heating parameters, which may provide some constructive suggestions for choosing the best experimental parameters in ionospheric modulated heating at mid-low latitudes in the future.

2. The theoretic model for modulated HF heating of the lower ionosphere

When modulated HF waves are transmitted into the ionosphere, the electron temperature in the disturbed region is enhanced, accompanied by changes in electron collision frequencies and perturbations of electric conductivities. By switching on-off heating modes, oscillating currents are

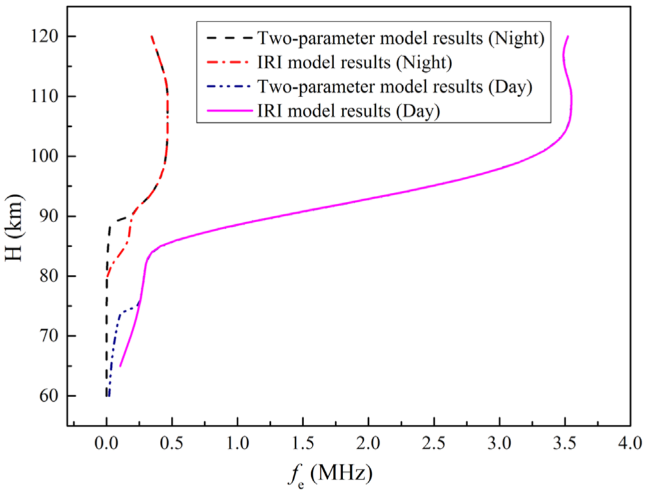

Figure 1. The plasma frequency profile in the lower ionosphere at Qingdao given by IRI and two-parameter model at noon (day) and midnight (night) on 15 October 2016.

generated with their eigenfrequency equal to the modulation frequency, and the resultant VLF/ELF waves may travel downwards into the Earth-ionosphere waveguide and upwards into the magnetosphere, respectively. Physical models for modulated HF heating of the ionosphere have been presented previously [26–29] and are not repeated here.

The electron energy equation is given by [26–29]

$$\frac{3}{2}\left(KN_e\frac{\partial T_e}{\partial t}\right) = Q - L \quad (1)$$

where N_e and T_e are the electron density and temperature in the ionosphere, respectively, K is the Boltzmann constant. Q is the unit energy absorbed by electrons from HF pumping wave per volume, and L is the unit energy loss caused by collision of electrons with neutral particles per volume.

The relationship of conductivity to electron temperature is [29]

$$\Delta\sigma_H = -2\frac{N_e e}{B}\frac{v_e w_e{}^2}{(v_e{}^2 + w_e{}^2)^2}\cdot\frac{dv_e}{dT_e}\cdot\Delta T_e \quad (2)$$

$$\Delta\sigma_P = \frac{N_e e}{B}\frac{w_e(w_e{}^2 - v_e{}^2)}{(v_e{}^2 + w_e{}^2)^2}\cdot\frac{dv_e}{dT_e}\cdot\Delta T_e \quad (3)$$

where σ_H and σ_P are Hall and Pedersen conductivity, respectively. $\Delta\sigma_H$ and $\Delta\sigma_P$ are variations of σ_H and σ_P during the heating period. ΔT_e is variation of electron temperature. e is electron charge, w_e is electron gyro-frequency, B is intensity of the geomagnetic field, and v_e is electron-neutral collision frequency.

The VLF/ELF radiation current ΔJ generated by modulated heating of the ionosphere can be expressed as

$$\Delta J = \mathbf{E}_0\cdot(\overrightarrow{\Delta\sigma_H} + \overrightarrow{\Delta\sigma_P}) \quad (4)$$

where $\mathbf{E}_0$ is the background natural electric field. As shown by equation (4), ΔJ is proportional to $\mathbf{E}_0$ which is dominantly affected by the natural ionosphere. Noted that $\mathbf{E}_0$ is not subject to artificial adjustments unlike the HF transmitting parameters such as the heating frequency and duty cycle. For simplicity, the influences of its variances on ΔJ are omitted in this paper. A constant empirical value is assumed throughout the D-region ionosphere and chosen to be 0.5 mV m^{-1} at mid-low latitudes.

We shall make improvements in this model according to the specific analysis in the next context and not repeated here. The simulation parameters are chosen as follows. The density and temperature of the neutral atmosphere are given by NRLMSIS00. Only two positive ions, NO^+ and O_2^+ are considered. While the other background ionosphere parameters are given by International Reference Ionosphere 2015 (IRI-2015), the electron density is modeled using a two-parameter exponential decline model [30], which is more suitable for a lower ionosphere less than 100 km compared with the IRI model and can be written in the following form:

$$N_e(h) = N_0 e^{-0.15h'}e^{(\beta-0.15)(h-h')} \quad (5)$$

where $N_e(h)$ is the electron density at the height of h and $N_0 = 1.43\times10^{13}$ m^{-3}, h' is the minimum height at which the electron density can be accurately predicted by the IRI model. β represents the profile steepness. In our simulation, the heating facility is supposed to be located at Qingdao (36°N, 120°E) with the geomagnetic latitude (~29°) comparable to that at Arecibo. h' and β are taken to be 75 km and 0.39 km^{-1} at daytime respectively [31] and chosen to be 90 km and 0.63 km^{-1} at nighttime [32]. As shown in figure 1, the plasma frequency profile (f_e) as a function of height in the lower ionosphere ranged from 65–120 km at 12:00 UT (Noon) and 24:00 UT (midnight) on 15 October 2016 are predicted by IRI model and the two-parameter model, respectively. It is clear that f_e deduced from the IRI model is appreciably higher than that given by the two-parameter model at the same altitude. The maximum of f_e at nighttime is quite low and less than 0.5 MHz comparative to the daytime value.

The lower ionosphere is gridded with an equal interval of 1 km. The square wave commonly used in modulated heating experiments and theoretical simulations is taken as the AM waveform here and the modulated depth is 1. The polarization of the HF pumping wave is O-mode or X-mode, both with an ERP of 100 MW.

3. Numeric simulation and analysis

3.1. Influence of the duty cycle on VLF/ELF radiation efficiency

In the square wave amplitude modulation experiments, duty cycle (D) refers to the fraction of the VLF/ELF period during which modulated heating occurs in the total modulation period. The relationship of D with the modulated power A_r can be expressed in the following form:

$$A_r(t) = D\sum_n P_{T/2}(t - T/2 - nT_1)$$
$$= D\left[1 + 2\sum_{k=1}\text{sinc}(k\omega_1 T/2)\cos k\omega_1 t\right] \quad (6)$$

where $P_a(x)$ is square wave delta function. If $|x|\leqslant a$, $P_a(x) = 1$ and if $|x| > a$, $P_a(x) = 0$. T_1 is modulation period

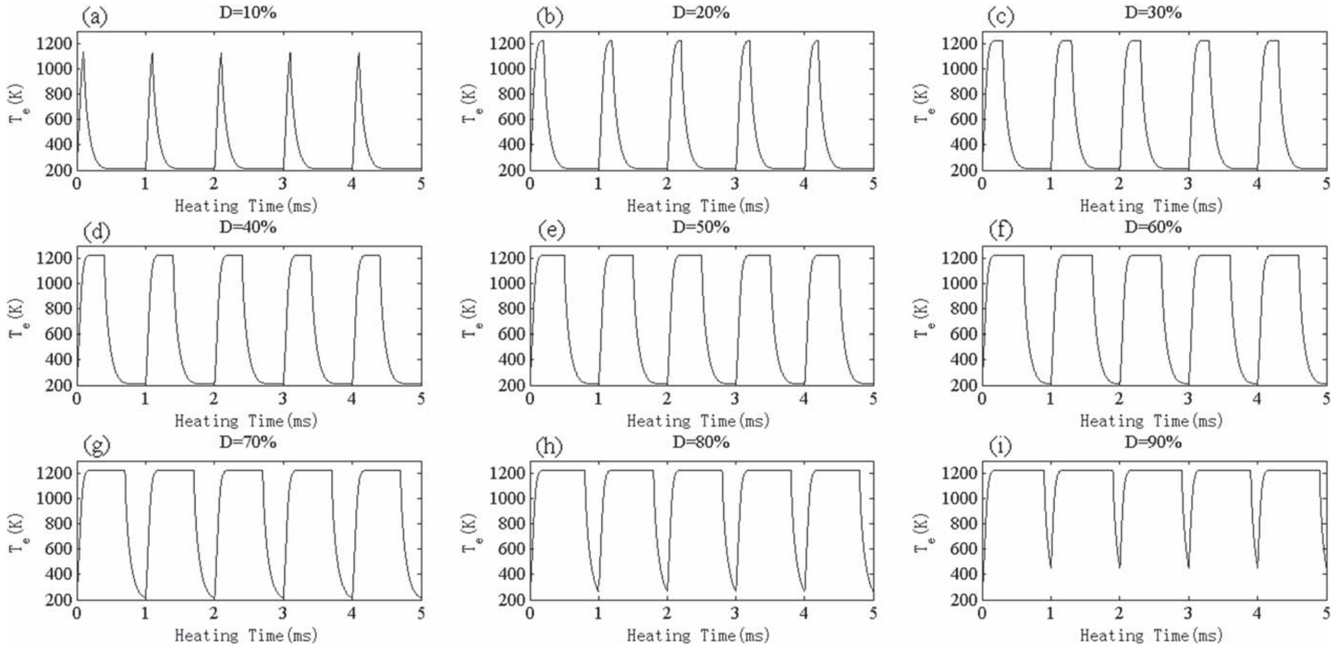

Figure 2. Variations of electron temperature with the heating time at different duty cycles ($H = 75$ km).

(VLF/ELF period), T stands for the on portion of the VLF/ELF period and thus $D = T/T_1$. ω_1 is the modulation angular frequency.

Energy absorption in the heating period is closely related to duty cycle as shown by equation (6). Changes of duty cycle lead to variances of energy absorption and result in changes of electron temperature perturbations. Dependences of electron temperature at the altitude of 75 km ($H = 75$ km) with heating period are plotted in figure 2 with nine different duty cycles varied step by step from 10%−90%. The heating frequency is 4.0 MHz and the VLF/ELF modulation frequency is 1 kHz. Electron density is given by the two-parameter model (day) and the polarization of HF pumping wave is the X-mode. As shown by figure 2, the peak electron temperature is elevated with the increase of the duty cycle. Electron temperature cannot be saturated with a duty cycle lower than 20% in the temperature rise period and cannot recover back to ambient conditions with a duty cycle larger than 70% in the temperature cooling period. Nevertheless, with a duty cycle between 20% and 70%, it is enough for the electron temperature to rise up to a maximum of 1220 K and to be cooled to a minimum of 210 K.

Time constants of electron temperature at different heights are highly variable and even exceed 15 ms above 95 km. If the heating portion of the modulation period is too short, it is difficult to cause considerable temperature perturbations at high attitudes of the lower ionosphere. Variations of electron temperature with heating time at different heights in the lower ionosphere for the case of 50% duty cycle ($D = 50\%$) are shown in figure 3. It can be seen that electron temperature cannot be saturated over 85 km and has no changes especially above 105 km. Duty cycle corresponds to the heating portions of a certain modulation period and has different effects on ionospheric parameters such as electron temperatures at different heights. The generated VLF/ELF radiation originates from the total heating areas and the intensity of the equivalent radiation source which can be expressed as the dipole moment **M** is summed over heights. By comparison, variances of **M** as a function of the duty cycle under X- and O-mode wave polarization conditions are given in figure 4, with the background electron density determined by IRI and two parameters, respectively. The influences of duty cycle on **M** at daytime and nighttime are plotted at figures 4(a) and (b) respectively. Only results given by the two-parameter model are demonstrated at figure 4(b) due to lack of proper descriptions of electron density lower than 80 km in the ionosphere with the IRI model. As shown in figure 4(a), **M** firstly rises and then drops with the increase in duty cycle. For different ionospheric models, the optimal duty cycle is slightly different. When using the IRI model, the VLF/ELF dipole moment peaks at a duty cycle of 50%, with an optimized range of 40%−70%, which has a different optimal range of 30%–60% and approaches the maximum at a duty cycle of 40% by the two-parameter model. It lies in the fact that the two-parameter model predicts a lower density compared with the IRI model. The characteristic time of electron temperature τ_h can be written in an integral form $\tau_h = \int_{T_e} \left[\frac{2}{3KN_e}(Q - L)\right]^{-1} dT_e$ from equation (1), and is proportional to electron density. Thus τ_h derived from the IRI model is appreciably larger than that from the two-parameter model, which corresponds to a longer duty cycle in a single heating period for generation of expected VLF/ELF radiation. It can be clearly seen from figure 4(a) that the generated **M** with the IRI model is significantly greater than the value calculated by the two-parameter model at daytime, due to higher background electron density predicted by the latter as shown in figure 1, which indirectly confirmed that enhancement of electron density by preheating the ionosphere proposed by Milikh *et al* [20] may improve VLF/ELF wave

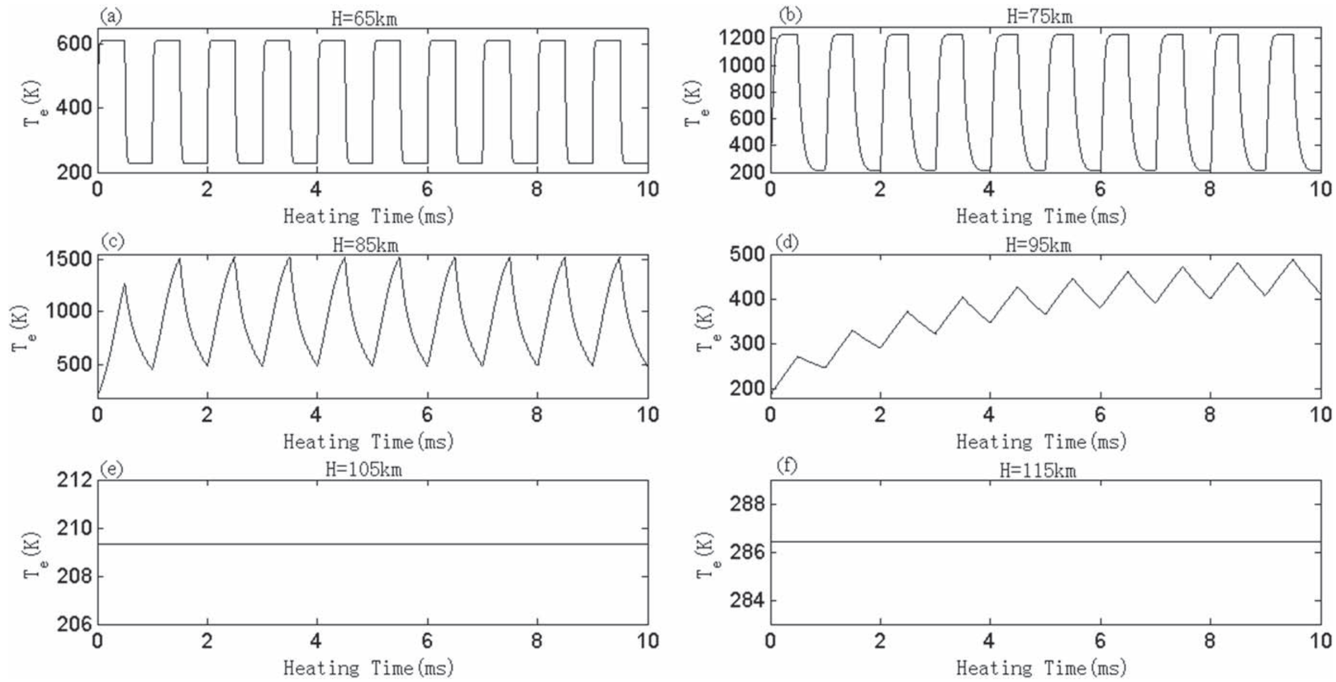

Figure 3. Variations of electron temperature with the heating time at different heights ($D = 50\%$).

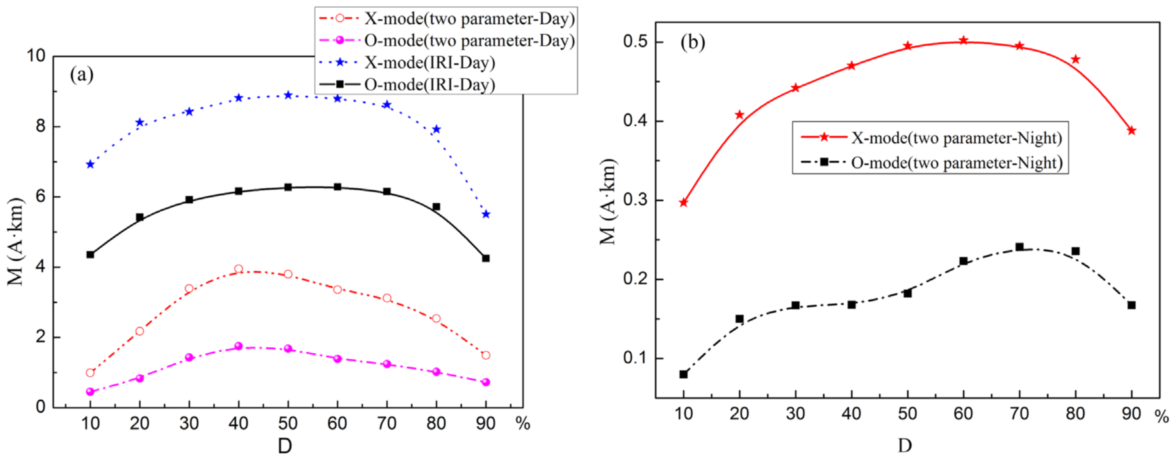

Figure 4. Variances of VLF/ELF dipole moment with duty cycle at daytime (a) and nighttime (b).

radiation. Similarly, the accordingly generated **M** at nighttime shown in figure 4(b) is much lower than that at daytime as shown in figure 4. Meanwhile, it is shown that although X-mode heating is more efficient than O-mode heating in VLF/ELF wave generation, the optimized range of duty cycle is almost independent of HF wave polarization.

3.2. Influence of heating frequency on VLF/ELF radiation efficiency

It has been suggested by Barr and Stubbe [33] that the equivalent dipole source of VLE/ELF radiation is located at a given height of 70–85 km in the ionosphere. It is well known that at a certain height, the Ohmic collision absorption energy of electrons decreases as the heating frequency increases for a fixed pumping power. Thus in order to obtain an ideal efficiency of VLF/ELF radiation, heating frequency is required to be matched with the local plasma frequency at a certain height of 70–85 km. However, the intensity of the equivalent VLF/ELF radiation source dipole moment is dependent on heating areas, which can be expressed in the following way:

$$\mathbf{M} = \mathbf{M}_{\mathrm{P}} + \mathbf{M}_{\mathrm{H}} = \sum_{i=1}^{n} \Delta\sigma_{i\mathrm{P}} \left(\frac{h_i}{h_0}\right)^2 A_0 \Delta h \mathbf{E}_0 + \sum_{i=1}^{n} \Delta\sigma_{i\mathrm{H}} \left(\frac{h_i}{h_0}\right)^2 A_0 \Delta h (\mathbf{E}_0 \times \mathbf{b}) \quad (7)$$

where $\mathbf{M}_{\mathrm{P}}$ and $\mathbf{M}_{\mathrm{H}}$ stand for the equivalent dipole moments corresponding to the Pedersen and Hall conductivities, respectively, n is the separated layers of the ionosphere, h_i is the height of the ith layer, and h_0 is the bottom height of ionosphere. A_0 is the bottom area illuminated by the heating wave beam. $\Delta\sigma_{i\mathrm{P}}$ and $\Delta\sigma_{i\mathrm{H}}$ are disturbances of σ_{P} and σ_{H} in

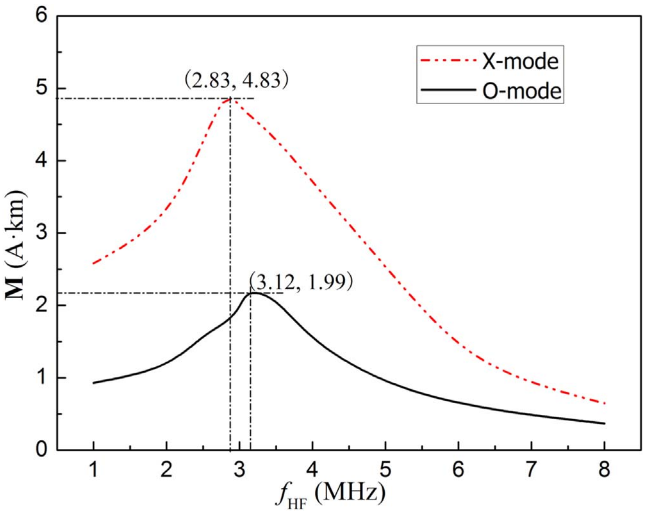

Figure 5. Effect of heating frequency on VLF/ELF radiation efficiency. The dashed red line indicates X-mode polarization and the solid black line indicates O-mode polarization.

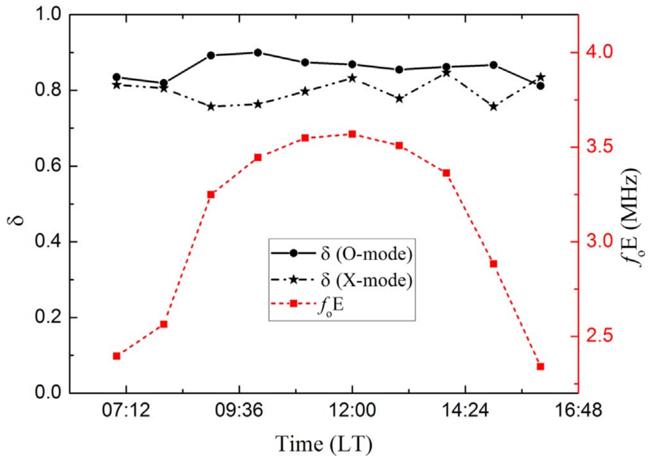

Figure 6. Variations of the critical frequency and the ratio of optimal heating frequency to the critical frequency with daytime.

the *i*th layer, respectively. **b** is the unit vector in the direction of the geomagnetic field.

As shown by equation (7), the intensity of VLF/ELF radiation source generated by modulated heating of the ionosphere stands for total effects in the perturbed region, which can be enhanced by an increase in perturbed height ranges and thus heating frequency, since HF waves with a greater heating frequency less than f_oF2 may reflect from a higher altitude, corresponding to a larger h_i in equation (7) and a perturbed disturbed region as a result. At night, the E layer densities drop by a factor of 100 or so, and the electron density in the D region is not directly observable by the conventional ionosonde and probably falls from about 10^3 per cm^3 at 80 km at noon to less than 10^2 per cm^3 at night, thus the f_oE is very small and may be less than 0.5 MHz as seen in figure 1, which is inaccessible for actual HF heating facilities. For example, the lowest heating frequencies at HAARP, EISCAT (European Incoherent Scatter Scientific Association), and Arecibo are 2.8, 3.85, and 3.0 MHz respectively. Thus heating frequency applied in active experiments may be much higher than the critical frequency of the lower ionosphere. The dipole moment **M** falls with increase of heating frequency in this context, as suggested by Milikh *et al* [24]. Consequently, only the daytime situation will be considered hereinafter. In figure 5, VLF/ELF dipole moment **M** is plotted as a function of heating frequency with the background electron density modeled by the two-parameter model. The duty cycle of the AM wave is 50% and the modulation frequency is 1 kHz. The dashed red line and the solid black line correspond to X-mode and O-mode heating waves, respectively. It is shown that the intensity of the generated VLF/ELF radiation source **M** firstly rises and then falls with the increase of heating frequency f_{HF} and drops rapidly once f_{HF} exceeds over the critical frequency f_oE. It is consistent with the results demonstrated in figure 4 that X-mode heating is more efficient than O-mode heating in VLF/ELF wave generation, due to more energy absorption. Therefore, the optimal X-mode frequency is appreciably less than for the O-mode for the same ambient ionosphere. Meanwhile, the heating frequency corresponding to the strongest VLF/ELF dipole is much higher than the plasma frequency at 70–85 km altitude. The relationship of the generated VLF/ELF radiation intensity with heating frequency has discrepancy with conclusions of an inverse proportion given by Milikh *et al* [24]. The difference can be explained in the context of f_{HF} compared with the critical frequency of the lower ionosphere, which is commonly referred to that of ionospheric E layer (f_oE) rather than that of the D layer. f_{HF} used by Milikh *et al* [24] is much larger than f_oE and has a narrow range, which hardly represents the real relationship of radiation efficiency with heating frequency. However, heating frequency has a wide range with consideration of the background ionosphere in this paper. It seems that the resultant conclusion herein has a comprehensive reflection of influences of f_{HF} on radiation efficiency. A similar relationship may be obtained if using the same frequency range as Milikh *et al* [24].

From the above analysis, optimal f_{HF} corresponding to the peak **M** is lower than f_oE and also changes with variations of f_oE. Since f_oE is closely related to the local solar illumination, it varies a greatly at different times. f_oE at daytime on 15 October 2016 given by the two-parameter model is displayed in figure 6. Typical variations of f_{HF} to f_oE ($\delta = f_{HF}/f_o$E) with time in the case of the strongest VLF/ELF radiation source generation are also plotted, which is applicable for the daytime context. To excite strong VLF/ELF radiation sources, δ ranges from 0.80–0.90 for the O-mode polarization and from 0.75–0.85 for X-mode heating case, which is clearly seen.

4. Discussion

The above research on the effects of duty cycle and heating frequency on VLF/ELF radiation intensity excited by ionospheric modulated heating are mainly aimed at the mid-low

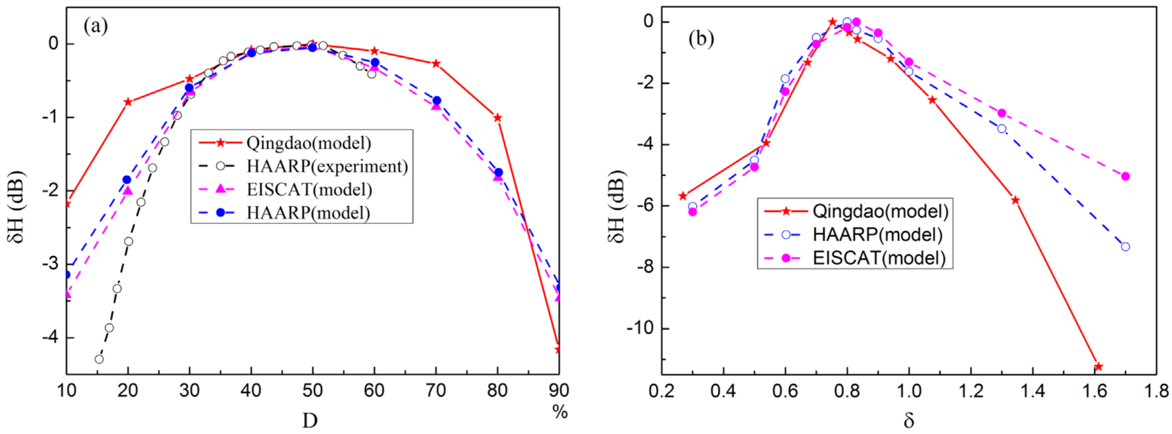

Figure 7. Comparison of the influence of D (a) and δ (b) in different latitudes.

latitudes. However, the ionospheric background varies with the geographical location. Therefore, in the relevant research in high latitudes, the above conclusions will be different. For example, Cohen *et al* [10] carried out square wave amplitude experiments using the HAARP heating device, it was found that the duty cycle corresponding to the maximum VLF/ELF was in the range of 30%–50%. In order to compare the difference of VLF/ELF radiation intensity between different D and δ in mid-low latitudes and high latitudes, Qingdao (36°N, 120°E), HAARP (62.4°N, 145.2°W), and EISCAT (69.6°N, 19.2°E) are selected as the contrast heating sites, of which HAARP and EISCAT are currently the main heating devices in high latitudes. The polarization of heating wave is X-wave, excitation frequency is 1 kHz, and the time is 12:00 on 15 October 2016. When analyzing the influence of duty cycle, the $\delta = 0.8$; when analyzing the influence of heating frequency, the $D = 50\%$. The comparison results are shown in figure 7. The ordinate in figure 7 shows the attenuation of the relative maximum amplitude of VLF/ELF under different conditions, expressed in δH.

Figure 7(a) shows that under the selected parameters, the maximum values of the three places appear in the range of 40%–55%. At the same time, the optimum range of duty cycle of Qingdao in the mid-low latitudes is obviously wider than that of HAARP and EISCAT in high latitudes, the former is 40%–70%, and the latter is 40%–60%. The HAARP experimental data (Chistochina reception, excitation frequency 990 Hz. Cohen *et al* [10]) are also added in figure 7(a), it is found that the theoretical simulation results agree well with the experimental results in the range of duty cycle greater than 30%. Figure 7(b) shows that the δ corresponding to the maximum VLF/ELF of the three regions of Qingdao, HAARP, and EISCAT increases sequentially. In other words, under the same parameters, the optimal δ value in the high latitude region is higher than that in the mid-low latitude regions, and the value increases with latitude, but the value is less than 1. Figure 7 also shows that the duty cycle and δ of HAARP and EISCAT at high latitudes have little different effect on VLF/ELF radiation.

5. Conclusion

With the modulated heating model on the basis of the theory of Ohmic heating of the lower ionosphere, we have studied influences of duty cycle and heating frequency on the efficiency of VLF/ELF generation via modulated HF heating of the lower ionosphere by numerical simulation and tried to analyze possible affecting factors. The conclusions are as follows.

(1) The optimized ranges of duty cycle for modulated heating of the ionosphere are closely related to the background D-region ionosphere. For the ambient electron density at daytime given by the two-parameter model, electron temperature at the altitude of 75 km cannot reach the maximum during a modulation period for the case of a square wave with a low duty cycle less than 20%. When the duty cycle is larger than 70%, electron temperature cannot be cooled timely. For the case with a 50% duty cycle, electron temperature at the altitude above 85 km cannot be saturated and remains almost constant above 105 km. The VLF/ELF dipole moment **M** firstly rises and then drops with the increase of duty cycle. Optimal duty cycle is slightly varied for different ionospheric models. When using the IRI model, **M** peaks at a duty cycle of 50%, optimally ranging from 40%−70%. For the two-parameter model case, an optimal duty cycle is 40% with an optimized range of 30%−60%. The generated **M** at nighttime is much lower than that at daytime. Although X-mode heating is more efficient than O-mode heating in VLF/ELF wave generation, the optimized ranges of duty cycle are almost independent of HF wave polarization.

(2) Optimized ranges of heating frequency are dependent on HF wave polarization.

The intensity of the VLF/ELF equivalent radiation source firstly rises and then falls with the increase of heating frequency. If heating frequency exceeds the critical frequency, the magnitude of **M** drops rapidly. For the same background ionosphere, the optimal X-mode heating frequency is appreciably less than that of the O-mode. The optimal f_{HF} may be 0.8–0.9 times of the f_oE for O-mode heating and 0.75–0.85 times for the X-mode case in order to obtain better VLF/ELF generation.

Finally, we compare the difference of D and δ effects between mid-low latitude and high latitude regions, and show that under the same parameters, the optimum range of D in mid-low latitudes is obviously wider than that in high latitudes, and the optimum value of δ in high latitudes is higher than that in mid-low latitudes, and the value increases with the increase of latitude.

It should be noted that influences of a natural electric field on VLF/ELF radiation efficiency have not been involved and a constant natural electric field has been assumed for us to obtain optimal heating frequency ranges in this paper. In practice, HF radio waves with higher frequency reach a higher level with a stronger reflected electric field, leading to a strengthened radiation source of VLF/ELF waves generated by modulated heating, which may affect optimal heating frequency ranges and will be considered in the next step.

The above-mentioned results may provide a reference for parametric choices to increase VLF/ELF radiation intensity for lower ionospheric AM heating experiments in the future. As the experimental abilities of the heating system are continuously strengthened and heating techniques progress, VLF/ELF signals generated by modulated heating are expected to be applicable and required to remain stable. By measuring background ionospheric states in real time with diagnostic devices, it is convenient to obtain the critical frequency of the lower ionosphere and adjust heating frequency timely according to optimal ratios of f_{HF} to f_oE so as to maintain intensity of VLF/ELF signals to a relatively high level.

Acknowledgments

This work is supported by the Innovation Fund of China Electronics Technology Group Corporation (No. KJ1602004).

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