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Influence of toroidal rotation on the tearing mode in tokamak plasmas

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Abstract

The stabilizing mechanism of toroidal rotation on the tearing mode is studied using the 3D toroidal resistive magnetohydrodynamic code M3D. It is found that the dominating mechanism, either the centrifugal effect or the Coriolis effect, depends on the specific pressure β and rotation frequency Ω. On the premise that Ω is sufficiently large, when β is greater than a critical value, the effect of the centrifugal force is dominant, and the stabilizing effect mainly comes from the modification of equilibrium induced by the centrifugal force; when β is less than a critical value, the stabilizing effect from the Coriolis force overcomes that from the centrifugal force. However, if Ω is small, then the effect of equilibrium modification due to the centrifugal force is not significant even if β is large. Finally, the results showed that toroidal rotation shear enhances the stabilizing effect.

Keywords: tearing mode, toroidal rotation, resistive MHD

(Some figures may appear in colour only in the online journal)

1. Introduction

The tearing mode (TM) is one of the most dangerous magnetohydrodynamic (MHD) instabilities, and can change the topology of the magnetic field, create magnetic islands, and increase local radial transport. Furthermore, the TM can produce a seed island, which triggers the neoclassical tearing mode (NTM). The NTM seriously influences plasma pressure and may cause disruption. Thus, suppressing the TM and understanding the mechanism are important for tokamak physics.

Neutral beam injection (NBI), as a primary auxiliary heating system in tokamaks, injects toroidal momentum and generates strong toroidal plasma rotation (in the following context, we use the shortened term 'rotation' to mean toroidal plasma rotation). It has been proven in experiments that rotation can affect MHD instabilities, including the TM [1, 2]. Experimental results of ASDEX Upgrade (the 'Axially Symmetric Divertor Experiment') showed that the onset threshold of the NTM increases with co- and counter-current direction rotation and with positive and negative rotation gradients [1]. However, in DIII-D, the experiments found that the effect of rotation on the NTM depends on the direction of the rotation [2]. Co-current rotation increases the 2/1 NTM onset threshold, while counter-current rotation does not have a significant effect on the NTM.

In the last two decades, there have been a number of theoretical and numerical studies on this topic [3–19]. In [3], the instability of the TM with rotation was obtained analytically, which showed that rotation plays a stabilizing role, and the stabilizing effect depends on the magnitude of the mach number M and adiabatic index Γ. Recently, the 3D MHD code CLT (Ci Liu Ti) was applied to simulate the effect of rotation on the TM in low-β plasmas [4]. The pressure $p(\psi)$ is modified by $p(\psi)\exp[m\Omega^2(R^2 - R_m^2)/2T]$ in order to study the effect of the equilibrium modification induced by the centrifugal force. In this simulation, the ψ modification induced by toroidal rotation is neglected. The result showed

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that rotation can stabilize the TM instability, in which the Coriolis effect plays a dominant role. Another numerical work using the NEAR code, which solves a set of reduced MHD equations, noted that the centrifugal force can produce a significant change in the equilibrium, and the modified equilibrium was generated by the TOQ code [5] via solving the Grad–Shafranov equation, while the Coriolis force was not involved. It was indicated that rotation causes a change in Δ' to stabilize the TM. There is disagreement regarding the rotation stabilizing mechanism between these two numerical studies. In this work, we will clarify the stabilizing mechanism of rotation on the TM.

This paper is organized as follows. In section 2, the simulation setup of the M3D [20] code is given. Section 3 shows the influence of rotation on the TM and its mechanism. Finally, conclusions are given in section 4.

2. Simulation setup

In this work, a set of resistive MHD equations is used in the M3D code:

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \tag{1}$$

$$\mathbf{J} = \nabla \times \mathbf{B}, \tag{2}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{3}$$

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{J} \tag{4}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \tag{5}$$

$$\rho \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \mathbf{J} \times \mathbf{B} - \nabla p + \mu \nabla^2 \mathbf{v}, \tag{6}$$

$$\frac{\mathrm{d}p}{\mathrm{d}t} = -\Gamma p \nabla \cdot \mathbf{v} + \rho \nabla \cdot \kappa \cdot \nabla \frac{p}{\rho}, \tag{7}$$

where $\mathbf{E}$, $\mathbf{J}$, $\mathbf{B}$, $\mathbf{v}$, η, ρ and p are the electric field, current density, magnetic field, plasma velocity, plasma resistivity, plasma density, and thermal pressure, respectively. In this resistive MHD model, $\mathbf{B}$ and $\mathbf{E}$ satisfy the simplified Maxwell's equation, and the fluid equations (5)–(7) are used to describe the plasmas. The variables are normalized as follows: $\mathbf{v} = v_0 \hat{\mathbf{v}}$, $v_0 \equiv \epsilon B_0 / \rho_0^{1/2} = \epsilon v_{\mathrm{A}}$, $\epsilon = a/R_0$, $\mathbf{B} = B_0 \hat{\mathbf{B}}$, $\rho = \rho_0 \hat{\rho}$, and $p = \epsilon B_0^2 \hat{p}$. Symbols with superscript ($\wedge$) are code variables. The simulation is carried out under a circular tokamak equilibrium with $\beta_0 = 1\%$, where β_0 is the toroidal beta value at the magnetic axis, and the aspect ratio is $R_0/a = 5.76$. The safety factor profile is given by $q(\psi) = 1.5 + 2\psi^2$, and the pressure profile is given by $p(\psi) = p_0 \exp(-2.5\psi)$, where ψ is the normalized poloidal flux. With finite resistivity, the TM is linearly unstable. Figure 1 shows the mode structure of the TM with $\eta = 2 \times 10^{-5}$, where $\delta C = -R\delta J_\varphi$ (δJ_φ is the perturbed current in the toroidal direction) and U is the plasma velocity stream function. It is a classical TM structure with $m/n = 2/1$ (m is the poloidal mode number, and n is the toroidal mode number). The scaling of γ with η is shown in figure 2, and the fitting of the scaling agrees well with the classical FKR (Furth–Killeen–Rosenbluth) [21] theoretical result.

3. Influence of rotation on the tearing mode

In this section, we investigate the influence of rotation on the TM and clarify the related mechanism. In the presence of rotation, the centrifugal force induces poloidal asymmetry of equilibrium profiles and the coupling of the magnetic curvature with the centrifugal and Coriolis forces provides a perturbed perpendicular current; correspondingly, a return parallel current is induced that affects the stability of the TM [3].

3.1. Equilibrium with rotation

Analytically, the centrifugal force inducing the poloidal asymmetry of equilibrium can be modelled by $p(\psi)\exp[\Gamma M^2(R^2 - R_0^2)/2]$, where $M = v_\varphi / v_{\mathrm{s}}$ is the Mach number and v_φ and $v_{\mathrm{s}} = (\Gamma p/\rho)^{1/2}$ are the toroidal plasma velocity and sound velocity, respectively [22]. For typical tokamaks, such as DIII-D [23], HL-2A [24] and ITER [25], the Mach number can be up to 0.2–1; thus, in this work, it will be in this range. In previous models, the magnetic flux surface and magnetic axis were assumed to be fixed. In this work, equilibrium with rotation is self-consistently calculated via a nonlinear relaxation process, and the process is shown in figure 3. In this process, the static equilibrium from the VMEC (Variational Moments Equilibrium Code) code is used for the initial profiles, and a rotation profile is added to the momentum equation with a rotation source. First, rotation is added to the system without resistivity ($\eta \simeq 0$). Due to the centrifugal force, the force balance is broken, and the kinetic energy (excluding the equilibrium rotational kinetic energy) grows to a peak value ($\gamma \simeq 0$), which means that a rough force balance is reached globally, and then the perturbed velocity is wiped. Ideally, after repeating this relaxation process many times (approximately 0.2 Alfvén times) the peak kinetic energy drops to a small value, which suggests that a new equilibrium is reached.

The modification of the poloidal flux ψ and safety factor q is shown in figure 4. Obviously, with high rotation speed the modification of the poloidal flux and q profile can be significant. It should be noted that the change in the q profile in the previous work [5] is quite different from this result. The reason is unclear and will be discussed in future work.

3.2. Influence of rotation on the tearing mode

In this section, with rotation, the modified equilibrium is considered, and both the Coriolis force and centrifugal force are included. The growth rate ratio γ/γ_0 (γ is the growth rate with rotation and γ_0 is the growth rate without rotation) versus $M_{q=2}$ is shown in figure 5, where $M_{q=2}$ is the Mach number at the $q = 2$ rational surface.

It can be found that rotation can significantly stabilize the TM when $\Gamma = 1.67$. By comparing the results with shear

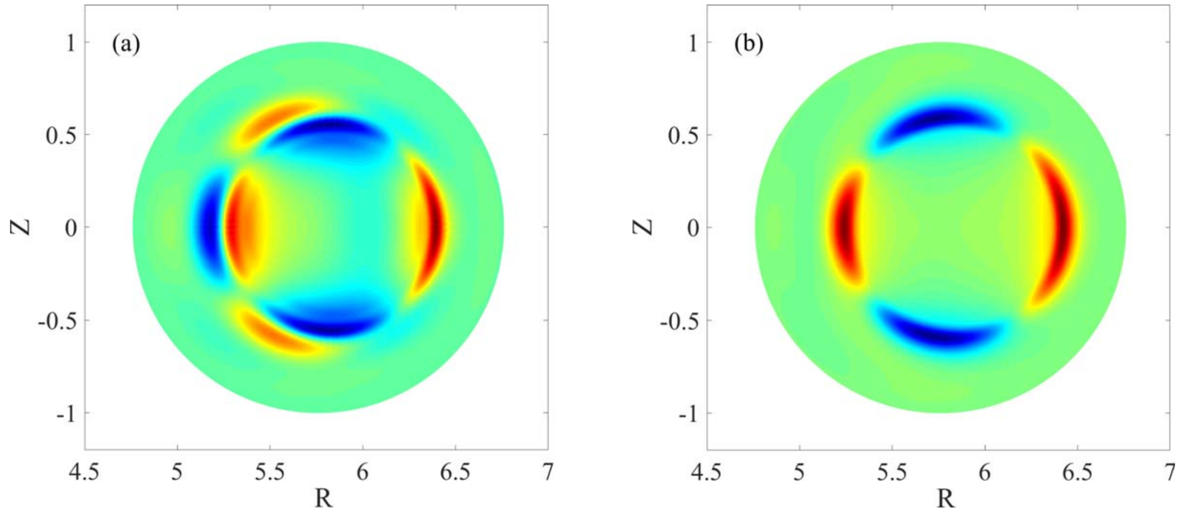

Figure 1. Tearing mode structure with $\eta = 2 \times 10^{-5}$: (a) $\delta C = -R\delta J_{\varphi}$ and (b) U.

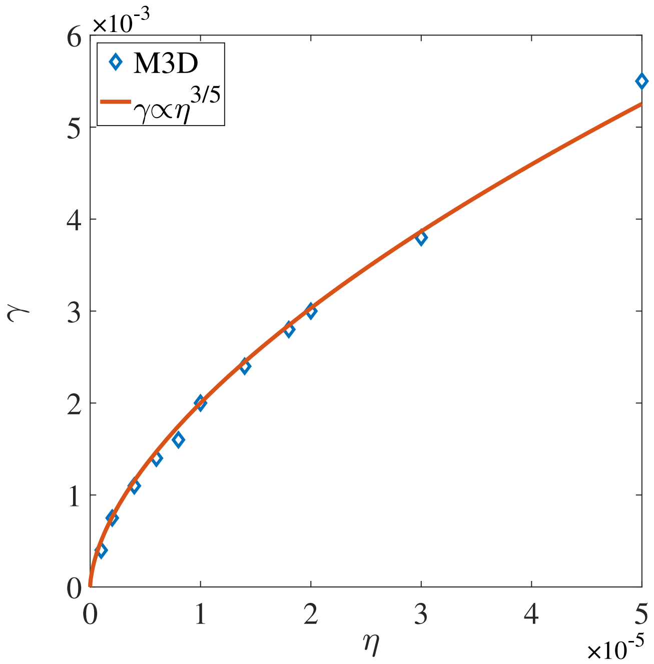

Figure 2. The scaling of γ with η.

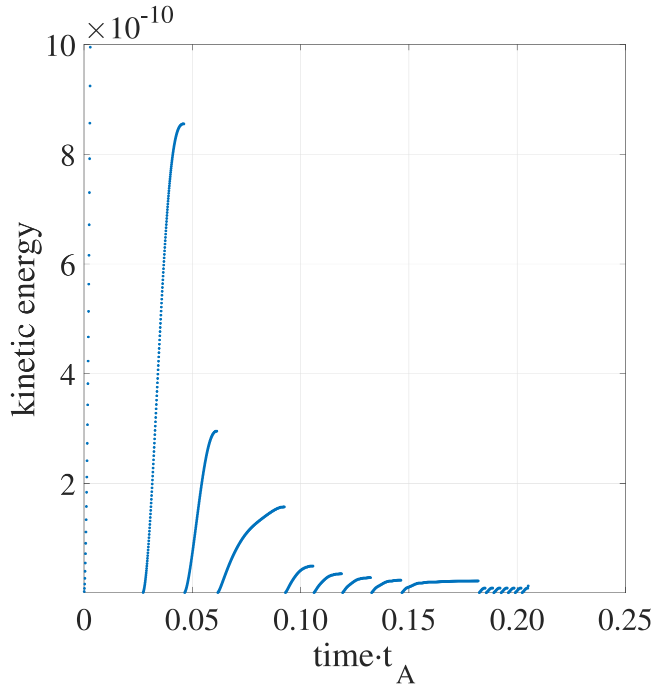

Figure 3. The kinetic energy with time in the nonlinear relaxation process.

(blue diamond) and without shear (red circle), it is found that the shear rotation profile can enhance the stabilizing effect. The results show that rotation stabilizing the TM can be divided into two stages. When the Mach number is small, the growth rate of the TM is slowly reduced. When the number is large, the growth rate rapidly and significantly decreases. By self-consistently considering the Coriolis force and centrifugal force, the simulation results agree well with the analytical results in [3] for low rotation speed. However, the simulation results exhibit a weaker stabilizing effect compared with the analytical results for high rotation speed. The reason for this could be that when the rotation is strong, the mode structure of the TM is broken, and the analytical model in [3] may be not applicable.

Next, the scaling of γ with η under rotation ($M_{q=2} = 0.18$) is analysed, and the results are shown in figure 6. Note that 0.18 is used as the Mach number in order to prevent the TM from being broken by strong rotation. The result shows that with $\Gamma = 1$ the scaling of the growth rate stays unchanged, and with $\Gamma = 1.67$ the scaling is changed to $\gamma \propto \eta$. The simulation results agree well with the theoretical conclusion in [3] for both $\Gamma = 1$ and $\Gamma = 1.67$, when the resistivity is small. When the resistivity is large, the scaling changes into $\gamma \propto \eta^{3/5}$, because the condition $M^4\beta/\gamma_0^2 \gg \hat{\gamma}^2$ is invalid.

Rotation can not only stabilize TMs but also change the mode structure. The classical TM is driven by a current, and in a circular tokamak, the mode structure is almost symmetrical.

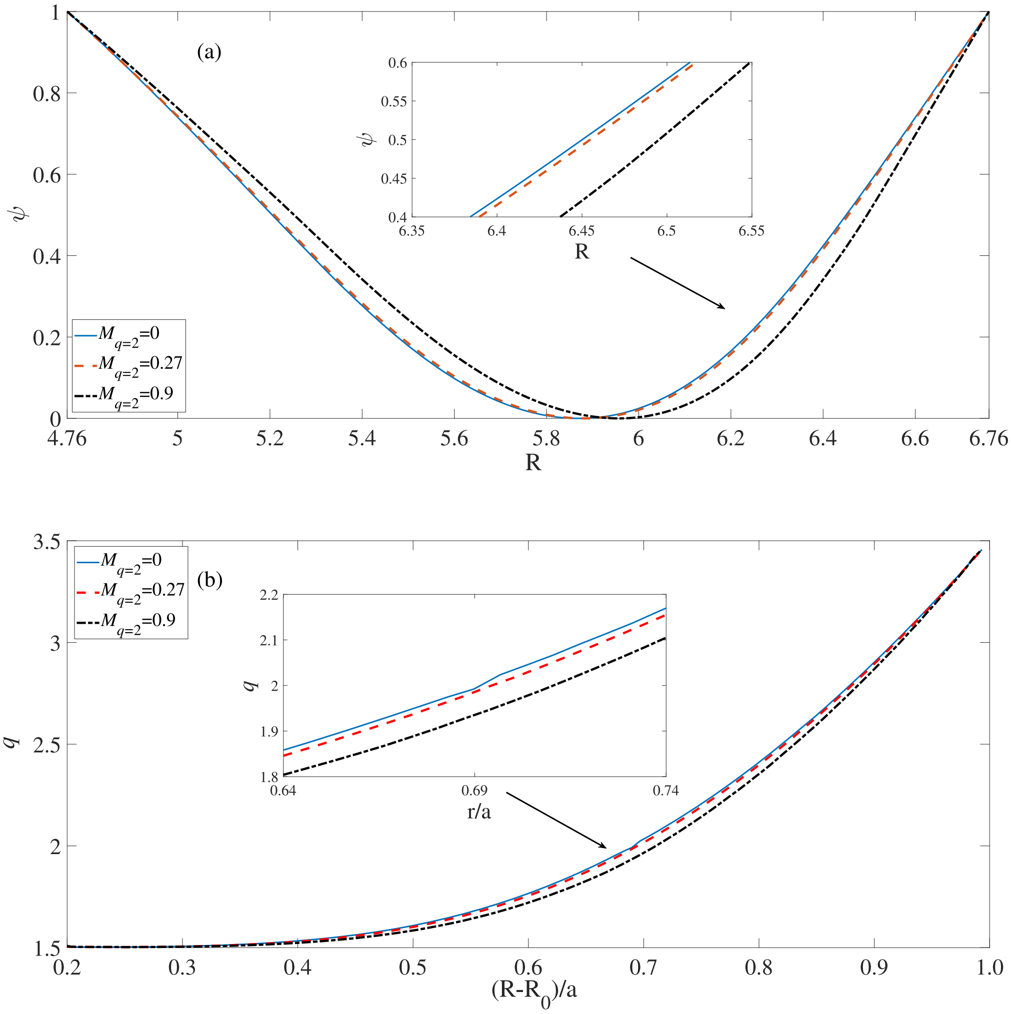

Figure 4. Modification of equilibrium profiles: (a) poloidal flux ψ and (b) safety factor q.

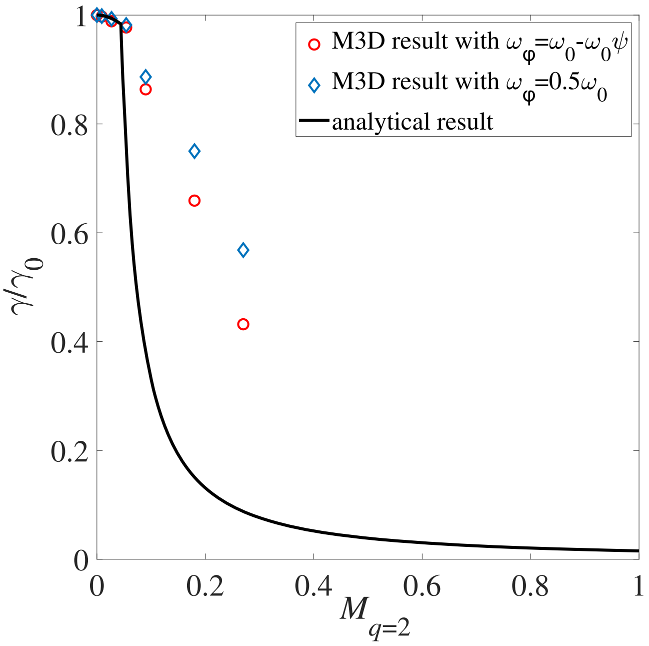

Figure 5. Comparison of growth rate ratio γ/γ_0 versus $M_{q=2}$ between the simulation and analytical results with $\Gamma = 1.67$.

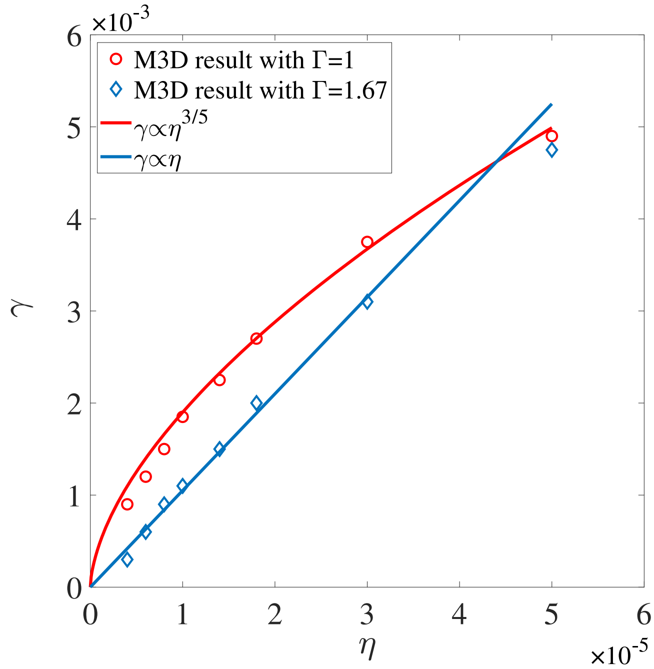

Figure 6. The scaling of γ with η when $M_{q=2} = 0.18$.

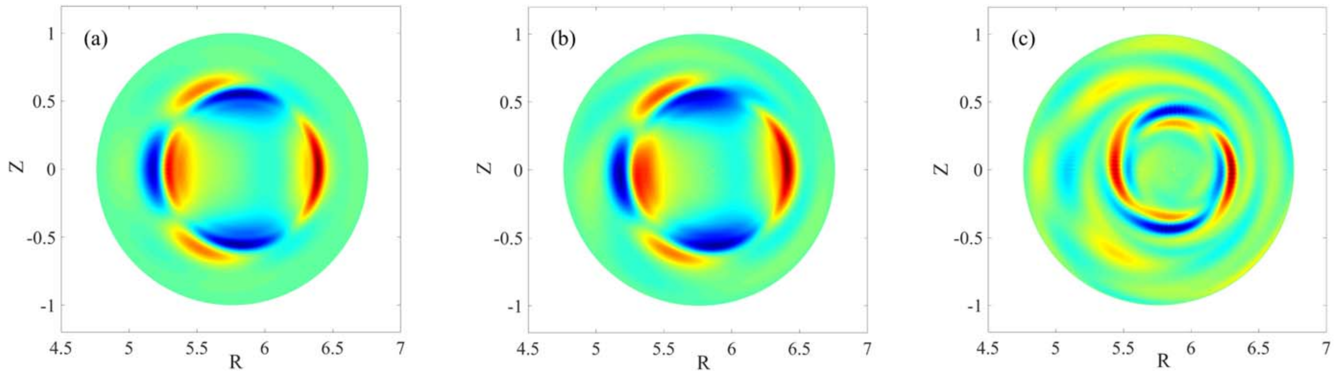

Figure 7. Tearing mode structures (δC) with different mach numbers at the $q = 2$ surface: (a) $M_{q=2} = 0$, (b) $M_{q=2} = 0.27$, and (c) $M_{q=2} = 0.54$.

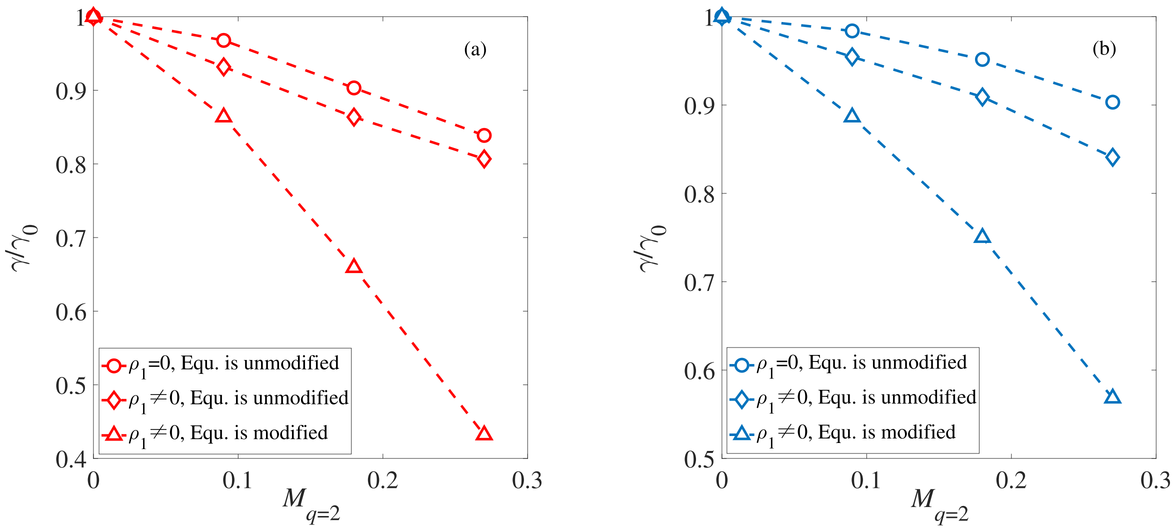

Figure 8. Growth rate ratio γ/γ_0 versus $M_{q=2}$ with $\beta_0 = 1\%$ and $\Gamma = 1.67$: (a) shear rotation profile and (b) constant rotation profile.

However, with high-speed sheared rotation, the mode structure is twisted and broadened, as shown in figure 7(c). Clearly, it is no longer a TM instability. This can be explained by the coupling effect of sheared rotation and viscosity [26, 27], which implies that the analytical model in reference [3] may be not applicable to cases with high-speed rotation. When the rotation speed is low, as shown in figure 7(b), rotation does not obviously change the TM mode structure.

3.3. Stabilization mechanism of rotation on the tearing mode

According to [3–5], the centrifugal and Coriolis forces induced by rotation are the key effects for rotation stabilization of the TM. In addition, the modification of the equilibrium generated by the centrifugal force is also important. Based on the linearized MHD equations with rotation, the perturbed centrifugal force and perturbed Coriolis force can be obtained: $-\rho_1 R\Omega^2\hat{\mathbf{e}}_R$ and $2\rho_0\Omega\hat{\mathbf{e}}_Z \times \mathbf{u}_1$, where $\mathbf{u}_1$ is the perturbed velocity [4, 28].

The stabilizing effects of rotation from the centrifugal force inducing an equilibrium shift, the perturbed centrifugal force and the perturbed Coriolis force are investigated, and the results are presented in figure 8. In this simulation, the Coriolis effect exists all the time, the perturbed centrifugal force can be controlled by turning on/off the time evolution of the density ρ_1, and the effect of equilibrium modification can be controlled by using the modified/unmodified equilibrium. In figure 8, $\rho_1 = 0$ means that the perturbation of the density is zero; the perturbed centrifugal force is then ineffective. In contrast, $\rho_1 \neq 0$ means that the perturbed centrifugal force is effective. When $\rho_1 = 0$ and the modification of the equilibrium is not considered, the stabilizing effects come from the perturbed Coriolis force alone. When $\rho_1 \neq 0$ and the modification of the equilibrium is not considered, the stabilizing effects come from both the perturbed Coriolis and perturbed centrifugal forces. When $\rho_1 \neq 0$ and the modification of the equilibrium is considered, the stabilizing effects come from the perturbed Coriolis force, perturbed centrifugal force, and equilibrium modification induced by the centrifugal force. By comparing the three lines in figure 8 for different situations and different Mach numbers, clearly, with finite Mach number, the stabilizing effects induced by rotation are non-negligible in any case. With small Mach number ($M_{q=2} < 0.1$), the centrifugal force inducing the equilibrium modification effect is comparable with the perturbed Coriolis force effect. However, with large Mach number ($M_{q=2} > 0.18$), the centrifugal force inducing equilibrium modification is the dominant factor, particularly in the sheared

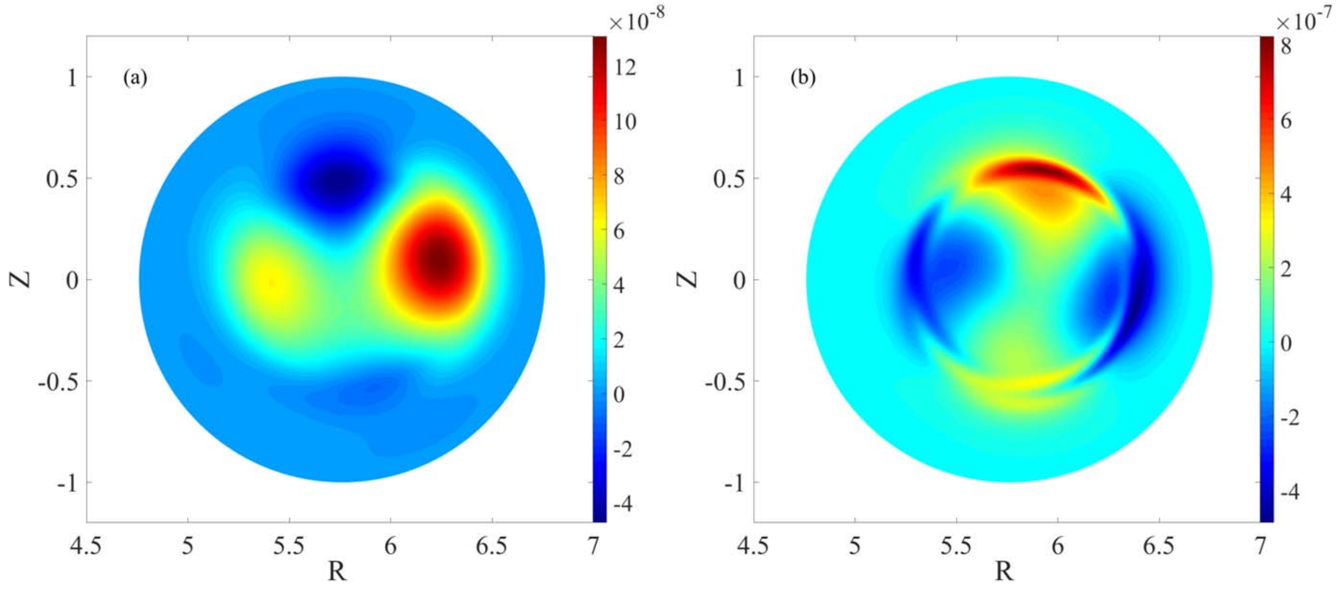

Figure 9. Magnitude of forces with $M_{q=2} = 0.27$ when equilibrium modification is not considered: (a) perturbed centrifugal force and (b) perturbed Coriolis force.

rotation profile cases shown in figure 8(a), and the stabilizing effects have the following relationship: equilibrium modification > perturbed Coriolis force > perturbed centrifugal force. Figure 9 shows the magnitude of the two perturbed forces with $M_{q=2} = 0.27$ when the modification of the equilibrium is not considered, and the perturbed centrifugal force is much smaller than the perturbed Coriolis force, which is consistent with the result above. Furthermore, the centrifugal force can induce a poloidal modification of the equilibrium, including the pressure and density profiles, which couples with the magnetic curvature to have a stabilizing effect on the TM. The results in figure 4 show that the equilibrium modification induced by the centrifugal force is small; however, it is the dominant factor for stabilizing the TM.

β also affects the equilibrium and thus impacts the TM, so the stabilizing mechanism with different β is studied. To investigate the influence of β on the stabilizing mechanism, the rotation frequency Ω is kept fixed, and the results are shown in figure 10.

$[\gamma(\Omega = 0.015,\ \beta) - \gamma(\Omega = 0,\ \beta)]/\gamma_0$ reflects the effect of rotation, where $\gamma(\Omega = 0.015,\ \beta)$ is the growth rate for $\Omega = 0.015$ rotation with β and $\gamma(\Omega = 0,\ \beta)$ is the growth rate in the absence of rotation with the same β; a larger absolute value of $[\gamma(\Omega = 0.015,\ \beta) - \gamma(\Omega = 0,\ \beta)]/\gamma_0$ means a greater stabilizing effect. The blue diamonds represent the effect of the perturbed Coriolis on the TM. The red diamonds reflect the stabilizing effect from the perturbed Coriolis and centrifugal forces. The black diamonds include the perturbed Coriolis, perturbed centrifugal and equilibrium modification effects. By comparing the three lines in figure 10, when β is small ($\beta = 0.1\%$) the Coriolis effect has a greater stabilizing effect than the centrifugal force. When $\beta \simeq 0.3\%$, the centrifugal effect including equilibrium modification is comparable with the Coriolis effect. Furthermore, as β increases, the effect of equilibrium modification becomes significant, and it is greater than the Coriolis effect when β is greater than the critical value. Based on both figures 10 and 8, it is necessary to note that the effect of equilibrium modification is related to β and Ω. On the premise that the rotation is sufficiently large, an increase in β will enhance the stabilizing effect of the equilibrium modification, and this effect can overcome the Coriolis effect when β is sufficiently large. However, when Ω is small, even if β is large, the effect of equilibrium modification is still not significant. The stabilizing effect of rotation is produced by the pressure–curvature term in addition to the toroidal coupling [29]. When β and Ω are large, the modification of equilibrium, particularly the pressure profile, produces a greater stabilizing effect on the TM.

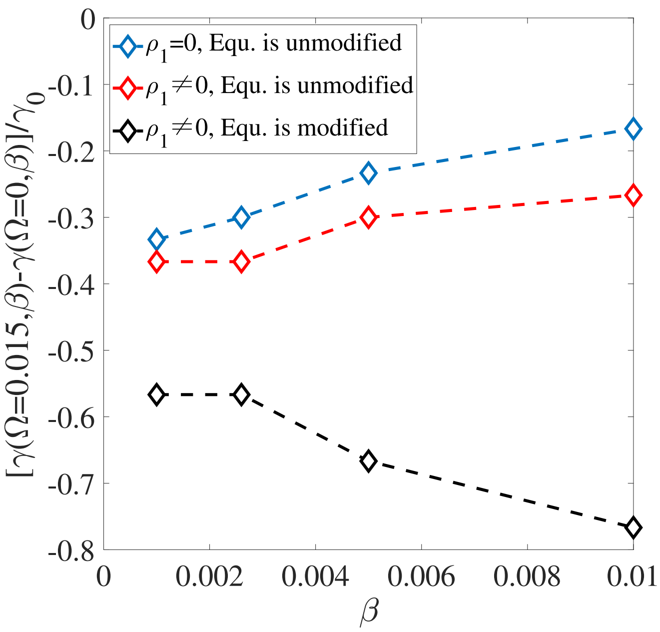

Figure 10. Growth rate ratio $[\gamma(\Omega = 0.015,\ \beta) - \gamma(\Omega = 0,\ \beta)]/\gamma_0$ versus β.

The results clarify the mechanism of rotation stabilizing the TM. The specific pressure β and rotation frequency Ω are two key parameters in the mechanism, which cause the different physical interpretations of the previous two simulation results in [4] and [5].

4. Conclusion

A global resistive MHD code (M3D) is used to study the influence of rotation on the $m/n = 2/1$ TM. The simulation results show that the TM mode structure is changed by sheared rotation, and strong sheared rotation ($M_{q=2} > 0.3$) significantly twists and broadens the classical TM structure. Rotation can stabilize the TM regardless of whether the rotation has shear; meanwhile, shear enhances the rotation's stabilizing effect. The coupling of the centrifugal and Coriolis forces with the magnetic curvature contributes to the stabilizing effect of rotation. In addition, the centrifugal force can induce poloidal asymmetry of the equilibrium profile, which changes the tearing mode instability. This work clarifies the stabilizing mechanism of rotation on the TM. The results show that the effect of equilibrium modification induced by the centrifugal force depends on β and the rotation frequency Ω. If Ω is sufficiently large, then increasing β will enhance the stabilizing effect of equilibrium modification.

(1) When β is greater than a critical value, the effect of the centrifugal force is dominant, and the stabilizing effect comes mainly from the equilibrium modification induced by the centrifugal force, although the equilibrium modification is small.

(2) When β is less than a critical value, the Coriolis force has a greater stabilizing effect than the centrifugal force.

However, if Ω is not sufficiently large, then the effect of equilibrium modification is not significant even if β is large.

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