The Self-Consistent Nonlinear Theory of Charged Particle Beam Acceleration by Slowed Circularly Polarized Electromagnetic Waves*

KONG Lingbao (孔令宝)[1], WANG Hongyu (王虹宇)[2], HOU Zhiling (侯志灵)[1], JIN Haibo (金海波)[3]

[1]School of Science, Beijing University of Chemical Technology, Beijing 100029, China
[2]School of Physics Science and Technology, Anshan Normal University, Anshan 114005, China
[3]School of Materials Science and Engineering, Beijing Institute of Technology, Beijing 100081, China

Abstract The relativistic interaction of charged particle beams with a circularly polarized electromagnetic wave propagating along a uniform guiding magnetic field in the tunneling of a dielectric medium is analyzed. The acceleration mechanism and a self-consistent nonlinear theory are presented for the interaction of relativistic charged particle beams with electromagnetic waves. Numerical results show that the beam particle can be efficiently accelerated in the interaction process.

Keywords: charged particle acceleration, slowed circular electromagnetic wave, self-consistent nonlinear equation, guiding magnetic field

PACS: *52.38.Kd*

DOI: 10.1088/1009-0630/15/12/02

1 Introduction

The study of relativistic charged particles with electromagnetic fields has fundamental importance in particle acceleration [1], plasma physics and controlled nuclear fusion [2,3], and the coherent generation or amplification of high-power microwaves [4,5], and has attracted interest from theoretical or experimental viewpoints for a long time. For example, BOURDIER et al. elegantly studied the interplay between a relativistic charged particle and a circularly or linearly polarized electromagnetic wave traveling in a vacuum along a guiding magnetic field through Hamiltonian formalism, and some interesting results and possible applications to particle acceleration or microwave amplification were suggested [6~10]. The nonlinear interactions of relativistic electrons with a circularly polarized electromagnetic wave (CPEW) in a guiding magnetic field were studied in Refs. [11~14] for the purpose of microwave generation. The acceleration of the electrons or ions by CPEW, intense laser pulses [15~18] or cyclotron waves in plasmas [19~25] were studied experimentally or numerically, and could have applications in plasma or accelerator physics. Most recently, Hamiltonian treatment to the relativistic particle-wave interaction had been presented in Ref. [26]. However, most of the previous studies on particle acceleration focus on nonlinear interactions between the particle and the electromagnetic wave in vacuum, or the electromagnetic wave contains a longitudinal component of the electric field. We will consider a rectilinear beam having an initial velocity along the propagation direction of slowed CPEW, and neglect the radiation of the particle beam. Then we will develop a self-consistent nonlinear theory for the interaction between charged particle beams and slowed CPEW. Numerical calculations are performed for the relativistic interaction of charged particle beams with a circularly polarized electromagnetic wave propagating along a uniform guiding magnetic field in the tunneling of a dielectric medium.

Our paper is constructed as follows. In section 2, we present the mechanism of charged particle acceleration by a slowed CPEW. The self-consistent nonlinear equations describing the interaction between charged particle beams and electromagnetic waves will be developed in detail. In section 3, we present numerical results for the relativistic factor and relative phase of the particle beam. A short conclusion is given in section 4.

2 Acceleration mechanism and general formulas

In order to illuminate the acceleration mechanism, we will consider a circularly-polarized electromagnetic wave with a constant amplitude propagating in a dielectric medium with refractive index n ($n > 1$) along a uniform guiding magnetic field $\mathbf{B_0}$ in the z direction. The electric and magnetic fields (including the guiding

*supported by National Natural Science Foundation of China (Nos. 51275029, 51102007 and 11275007)

magnetic field $\mathbf{B_0}$) of space can be written as

$$\mathbf{E} = E\cos(\omega t - kz)\,\mathbf{e_x} + E\sin(\omega t - kz)\,\mathbf{e_y}, \quad (1)$$

$$\mathbf{B} = -nE\sin(\omega t - kz)\,\mathbf{e_x} + nE\cos(\omega t - kz)\,\mathbf{e_y} + B_0\mathbf{e_z}, \quad (2)$$

in which E is the amplitude of the electric field, ω the angular frequency, $k = n\omega/c$ the wave number in the medium, n the refractive index of the medium, and $\mathbf{e_x}$, $\mathbf{e_y}$, $\mathbf{e_z}$ the unit vectors of the x-, y-, and z-axis, respectively.

Without loss of generality, we assume that the charged particle is an electron. The equations for electron motion and energy change in the above mentioned electromagnetic fields and guiding magnetic field $\mathbf{B_0}$ are

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = -e\left[\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B}\right], \quad (3)$$

$$\frac{\mathrm{d}W}{\mathrm{d}t} = -e\mathbf{v}\cdot\mathbf{E}. \quad (4)$$

Here, t is the time in the laboratory frame, $\mathbf{v}$ is the beam electron velocity, e is the charge of a positron, $\mathbf{p} = m_0\gamma\mathbf{v}$ is the electron momentum, m_0 is the rest mass of the electron, $\gamma = 1\Big/\sqrt{1 - v^2/c^2}$ is the relativistic factor, while $W = m_0\gamma c^2$ is the electron energy and c is the speed of light in vacuum.

We assume that the electron moves along the z-axis from the origin of the coordinate at initial time. Then the transverse force $\mathbf{F}_\perp$ acting on the electron due to the electric and magnetic field of the wave is

$$F_\perp = eE\left(\frac{v_z}{c/n} - 1\right). \quad (5)$$

Here, v_z is the electron longitudinal velocity. In the first interaction stage, $v_z < c/n$. Since $F_\perp$ is described by Eq. (5), the electron moves in the inverse direction of the electric field $\mathbf{E}$ and the transverse velocity $\mathbf{v}_\perp$ will be built up gradually, giving rise to an acceleration of electrons in the interaction process. Consequently, a longitudinal force F_z due to the transverse velocity and magnetic field is induced as follows:

$$F_z = \frac{neEv_\perp}{c}. \quad (6)$$

Because F_z is along the z-axis, the electron can be accelerated in the z-direction. The situation will be different when $v_z > c/n$, where the direction of $F_\perp$ is alternated and the transverse velocity $v_\perp$ of the electron will decrease. Besides, as suggested by Eq. (6), F_z does not change its direction and the longitudinal velocity can also be increased. According to Eq. (4), we have $\mathrm{d}\gamma/\mathrm{d}t = ev_\perp E/m_0c^2 > 0$, and therefore the electron can still be accelerated in this interaction stage.

From the practical viewpoint of charged particle beam acceleration, the energy coupling between the electron beam and the electromagnetic wave calls for amplitude variation of the wave. So the electric field in Eq. (1) must be modified as

$$\mathbf{E} = E(z)\cos(\omega t - kz)\,\mathbf{e_x} + E(z)\sin(\omega t - kz)\,\mathbf{e_y}. \quad (7)$$

In which $E(z)$ is the variant amplitude of the electric field. According to Faraday's law, the associated magnetic field in Eq. (2) can be rewritten as

$$\mathbf{B} = -\frac{n}{k}\frac{\partial}{\partial z}\mathbf{E}. \quad (8)$$

The electric field of the wave in Eq. (7) and the beam current are connected by the wave equation

$$\frac{\partial^2}{\partial z^2}\mathbf{E} - \frac{n^2}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{E} = \frac{4\pi}{c^2}\frac{\partial}{\partial t}\mathbf{j}_\perp, \quad (9)$$

in which $\mathbf{j}_\perp$ is the transverse current density of the beam. According to the current continuation equation, $\mathbf{j}_\perp$ can be presented as

$$\mathbf{j}_\perp = -\frac{n_0 e v_0}{v_z}\mathbf{v}_\perp, \quad (10)$$

where n_0 and v_0 are the initial beam density and beam electron velocity, respectively.

In order to simplify the self-consistent nonlinear equations that describe the electron beam and electromagnetic wave interaction, we introduce the following transformations and dimensionless parameters: $\mathbf{p} = m_0\gamma\mathbf{v} = p_\perp\cos(\omega t - kz + \Phi)\,\mathbf{e_x} + p_\perp\sin(\omega t - kz + \Phi)\,\mathbf{e_y} + p_z\mathbf{e_z}$, $\xi = kz$, $w = 1 - \frac{\gamma}{\gamma_0}$, $A = eE/m_0\gamma_0\omega c$, $\beta_0 = v_0/c$, $\gamma_0 = \left(1 - \beta_0^2\right)^{-1/2}$, $\Omega_0 = eB_0/m_0\gamma_0 c$, $p_\perp = m_0\gamma v_\perp$, $p_z = m_0\gamma v_z$, $p'_\perp = p_\perp/m_0\gamma_0 c$, $p'_z = p_z/m_0\gamma_0 c$, $\mu = \Omega_0/\omega$, $\omega_\mathrm{p}^2 = 4\pi n_0 e^2/m_0\gamma_0$, $I = \beta_0\omega_\mathrm{p}^2/2n\omega^2$. Here, Φ is the relative phase angle between the transverse velocity of the beam electron and the electric field of the wave.

Combining Eqs. (3), (4) and (7)~(10), we finally obtain the following self-consistent nonlinear equations for beam-wave interactions:

$$\frac{\mathrm{d}w}{\mathrm{d}\xi} = \frac{Ap'_\perp}{np'_z}\cos\Phi, \quad (11)$$

$$\frac{\mathrm{d}p'_z}{\mathrm{d}\xi} = -\frac{Ap'_\perp}{p'_z}\cos\Phi - \frac{I{p'_\perp}^2}{2n(p'_z)^2}\sin 2\Phi, \quad (12)$$

$$\frac{\mathrm{d}p'_\perp}{\mathrm{d}\xi} = \left(1 - \frac{1-w}{np'_z}\right)A\cos\Phi + \frac{Ip'_\perp}{2np'_z}\sin 2\Phi, \quad (13)$$

$$\frac{\mathrm{d}\Phi}{\mathrm{d}\xi} = 1 + \frac{\mu + w - 1}{np'_z} - \left(1 - \frac{1-w}{p'_z}\right)\frac{A\sin\Phi}{p'_\perp} + \frac{I\cos^2\Phi}{np'_z}, \quad (14)$$

$$\frac{\mathrm{d}A}{\mathrm{d}\xi} = \frac{Ip'_\perp}{np'_z}\cos\Phi. \quad (15)$$

The self-consistent nonlinear Eqs. (11)~(15) provide an efficient tool to study the interaction between an electron beam and electromagnetic wave along the constant guiding magnetic field in a tunnel inside the media.

3 Numerical simulations

The self-consistent nonlinear Eqs. (11)$\sim$ (15) can not be solved analytically. However, these are a couple of ordinary differential equation sets and will be solved using the self-adaptive Runge-Kutta method. Since we assume that the initial electron velocity is along the z-axis, it is obvious that the initial conditions for Eqs. (11)$\sim$ (15) can be presented as $w|_{\xi=0} = 0$, $p'_z|_{\xi=0} = \beta_0$, $p'_\perp|_{\xi=0} = 0$, $A|_{\xi=0} = \frac{eE(0)}{m_0\gamma_0\omega c}$, and $\Phi|_{\xi=0} = \pi$. It is worth noticing that with the help of relation $\mathrm{d}/\mathrm{d}t = kv_z\mathrm{d}/\mathrm{d}\xi$, the variable ξ in Eqs. (11)$\sim$ (15) can be transformed easily to the interaction time t. In the following, some typical quantities will be presented as functions of time t.

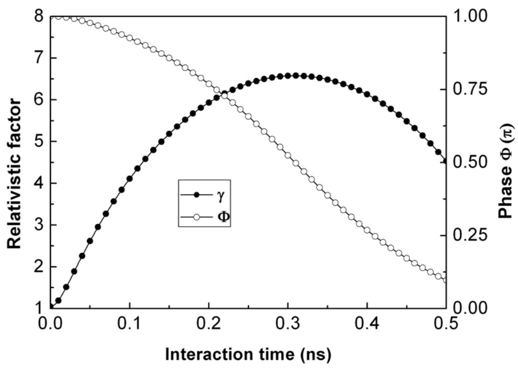

Fig.1 The relativistic factor γ and the relative phase Φ vs. laboratory time t. Here, $n_0 = 2 \times 10^9$ cm^{-3}, $n = 1.1$, $E = 5000$ statvolt/cm, the guiding magnetic field $B_0 = 1000$ Gauss, $\gamma_0 = 1.0494$, and the wave frequency $f = 4.0\times10^9$ Hz

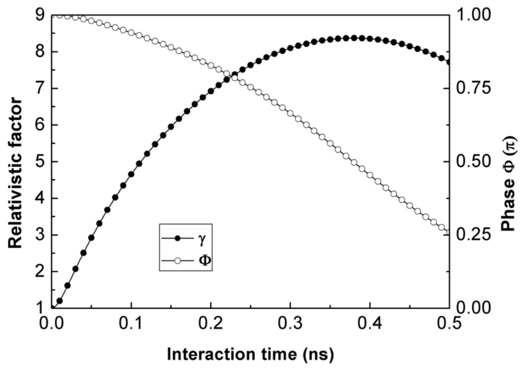

Fig.2 The relativistic factor γ and the relative phase Φ vs. laboratory time t. Here, γ_0=1.0146, and the wave frequency f=3.0$\times$10^9 Hz. The other interaction parameters are the same as in Fig. 1

In Fig. 1 and Fig. 2, the relativistic factor and phase as functions of t are given. For both lines, we take $n_0 = 2 \times 10^9$ cm^{-3}, $n = 1.1$, $E = 5000$ statvolt/cm, and the guiding magnetic field $B_0 = 1000$ Gauss; the wave frequency f and initial relativistic factor γ_0 are 4.0×10^9 Hz, 1.0494, and 3.0×10^9 Hz, 1.0146 for Fig. 1 and Fig. 2, respectively. From Fig. 1 and Fig. 2, we can see that the maximum of the relativistic factor and the corresponding time t (ns) are, respectively, 6.5769, 0.309, and 8.3719, 0.379. One can also see that the time for the phase to reach $\Phi = \pi/2$ is equal to the time for the relativistic factor to reach maximum. The results agree well with the theory, in which the beam electrons are in the accelerating phase when $\pi/2 \le \Phi \le \pi$, and the maximum relativistic factors approach $\Phi = \pi/2$.

In Fig. 3 and Fig. 4, the curves of coordinates x and y vs. laboratory time t are presented, and the corresponding interaction parameters are the same as those in Fig. 1 and Fig. 2. From these two figures, the particle trajectory can be recognized as a helix-like line. When the particle is running along the z axis, it rotates and drifts in the x and y directions. Then the particle exchanges energy with the electromagnetic wave, and the particle can interchange its kinetic energy between transverse and longitudinal movements. We can see that for a higher frequency, the drifting distance x or y is smaller, so we may choose a higher wave frequency in practical applications for charged particle beam acceleration in the tunneling of a dielectric medium.

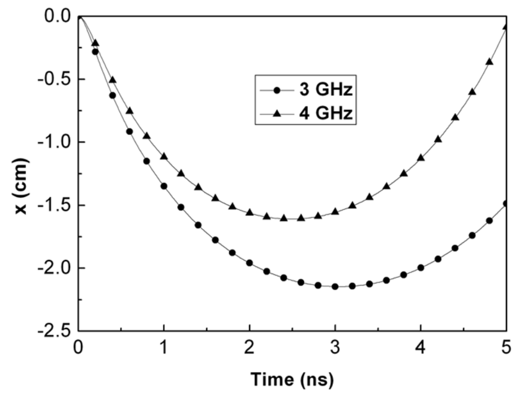

Fig.3 The x coordinate vs. laboratory time t. Here, the corresponding interaction parameters are the same as those used in Fig. 1 and Fig. 2

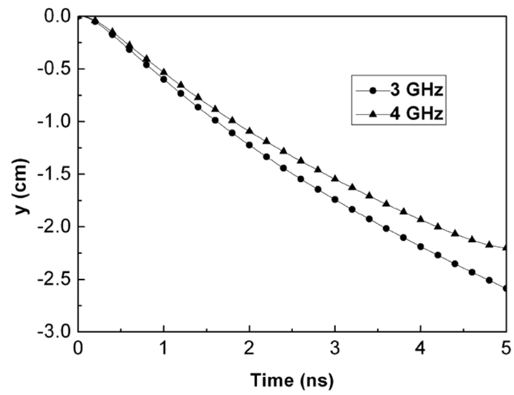

Fig.4 The y coordinate vs. laboratory time t. Here, the corresponding interaction parameters are the same as those used in Fig. 1 and Fig. 2

The numerical results also show that the peak relativistic factor (the line with the solid circle) in Fig. 2 is higher than that in Fig. 1. This is due to the higher wave frequency in the first case. Consequently, in the case of a lower wave frequency, the time for the relative angle Φ to approach $\pi/2$ becomes longer. Therefore, more energy can be transferred from the electromagnetic wave to the electron, leading to a more efficient acceleration of the electron in the interaction.

4 Conclusion

We analyzed the mechanism of the rectilinear charged particle beam acceleration by a CPEW propagating along a uniform guiding magnetic field in a tunnel inside the media with a refractive index of $n > 1$. The self-consistent nonlinear equations which describe the interaction between the beam and the electromagnetic wave were presented. Numerical results show that the particle can be efficiently accelerated. This study may have potential applications to charged particle acceleration for the purpose of plasma or accelerator physics.

References

1 Davidson R C, Qin H. 2001, Physics of Intense Charged Particle Beams in High Energy Accelerators. Imperial College Press, London
2 Jaroszynski D A, Bingham R, Cairns R A. 2008, Laser-Plasma Interactions. CRC Press, New York
3 Atzeni S, Meyer-ter-Vehn J. 2004, The Physics of Inertial Fusion: Beam Plasma Interaction, Hydrodynamics, Hot Dense Matter. Oxford University Press, New York
4 Kartikeyan M V, Borie E, Thumm M K A. 2004, Gyrotrons: High-Power Microwave and Millimeter Wave Technology. Springer, Berlin
5 Tsimring S E. 2007, Electron Beams and Microwave Vacuum Electronics. Wiley, New Jersey
6 Bourdier A, Gond S. 2001, Phys. Rev. E, 63: 036609
7 Bourdier A, Gond S. 2000, Phys. Rev. E, 62: 4189
8 Bourdier A. 2009, Physica D, 238: 226
9 Bourdier A, Patin D, Lefebvre E. 2005, Physica D, 206: 1
10 Bourdier A, Drouin M, Davoine X. 2010, IEEE Transactions on Plasma Science, 38: 728
11 Kong L B, Hou Z L, Xie C R. 2011, Appl. Phys. Lett., 98: 261502
12 Kong L B, Liu P K, Xiao L. 2007, Phys. Plasmas, 14: 053108
13 Kong L B, Liu P K. 2007, Phys. Plasmas, 14: 063010
14 Kong L B, Du C H, Liu P K, et al. 2007, J. Appl. Phys., 102: 103305
15 Pukhov A, Sheng Z M, Meyer-ter-Vehn J. 1999, Phys. Plasmas, 6: 2847
16 Wong L J, Kärtner F X. 2011, Appl. Phys. Lett., 99: 211101
17 Sharma R P, Chauhan P K. 2008, Phys. Plasmas, 15: 063103
18 Dodin I Y, Fisch N J. 2008, Phys. Plasmas, 15: 103105
19 Geyko V I, Dodin I Y, Fisch N J, et al. 2010, Phys. Plasmas, 17: 023105
20 Tsiklauri D. 2011, Phys. Plasmas, 18: 092903
21 Lapenta G, Markidis S. 2011, Phys. Plasmas, 18: 072101
22 Huang C, Lu Q M, Wang S. 2010, Phys. Plasmas, 17: 072306
23 Kneip S, Nagel S R, Martins S F, et al. 2009, Phys. Rev. Lett., 103: 035002
24 Araneda J A, Maneva Y, Marsch E. 2009, Phys. Rev. Lett., 102: 175001
25 Kallos E, Katsouleas T, Kimura D W, et al. 2008, Phys. Rev. Lett., 100: 074802
26 Zhmoginov A I, Dodin I Y, Fisch N J. 2011, Phys. Lett. A, 375: 1236

(Manuscript received 26 December 2012)
(Manuscript accepted 7 March 2013)
E-mail address of corresponding author
WANG Hongyu: why@btitgroup.com
E-mail address of KONG Lingbao:
konglingbao@gmail.com