

Tunability of One-Dimensional Plasma Photonic Crystals with an External Magnetic Field*

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Abstract We investigated in detail how photonic band structures (PBSs) of one dimensional plasma photonic crystals (PPCs) are tuned after being exposed to an external magnetic field. We showed that the properties of PBSs of PPCs are tuned correspondingly because the dielectric constant of the micro plasma layer is modified differently in different frequency ranges due to magneto-optical effects. Two numerical cases are calculated and discussed to study the magneto-optical effects on the properties of PBSs, including the Faraday and Voigt effects.

Keywords: photonic band structures, magneto-optical effects, Faraday effect, Voigt effect

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1 Introduction

Photonic crystals (PCs) are artificial structures with periodically modulated dielectric constants, in which there appear forbidden frequency regions—the so called photonic band gaps. Since the pioneering works of YABLONOVITCH^[1] and JOHN^[2] on this field many new inquisitive ideas have been developed. Recently, a new emerging area in the study of PCs, periodic plasma structures named plasma photonic crystals (PPCs) have received attention, and pertinent research activities have been extended^[3~14]. PPC is a composite structure with a periodic arrangement of plasma and dielectric material which has unique properties compared with conventional PC composed of dielectrics or metals. PPC represents a new type of tunable PCs, which can serve as an excellent tool to manipulate EM wave propagation, leading to many interesting phenomena as well as important applications. Moreover, by replacing solid material with plasmas, two important features are added to the usual PCs: time-varying controllability and a strong dispersion around the plasma electron density. These facts will lead to EM waves ranging from microwaves to THz waves, according to the scale and electron density of such plasmas^[3].

One dimensional PPC is an artificial periodic array composed of plasma and dielectric materials^[3~16] or plasma with periodic density distribution^[17,18]. Recently, several studies have been performed both analytically and numerically. HOJO et al.^[4] have investigated the dispersion relationship versus plasma density and plasma thickness using a method analogous to Kronig-Penny's problem in quantum mechan-

ics. LAXMI et al.^[5] have studied the photonic band gap effects in one dimensional plasma dielectric photonic crystals. Both SAKAI et al.^[6,7] and LIU et al.^[8] have investigated the collision effect of EM waves propagating in PPC. The former studied two dimensional PPC and the latter studied one dimensional PPC. But neither of the above-mentioned two research groups has investigated the oblique incidence effects. So in our previous papers^[9,10], we have discussed the properties of PBSs of one dimensional PPC for both normal and oblique incidences with collision effect. YIN et al.^[11] investigated theoretically and numerically the band gap characteristics of a one dimensional PPC composed of plasma layers of different densities. All the works mentioned above focused on the photonic band gap by changing the plasma parameters, dielectric constant and filling factor. No external magnetic field was considered until ZHANG et al.^[12] studied the transmission properties of a normal incident Gauss impulse in a one dimensional magnetized PPC by the finite-difference time-domain (FDTD) method. Recently, QI et al.^[13,14] studied the properties of an EM wave in a one dimensional magnetized PPC. This study showed that due to the magneto-optical effects, the dielectric constants of the plasma layer are modified differently in different frequency ranges. As a result, the PBSs of the PPC are tuned correspondingly. Consequently, to obtain a tunable PC which can be used to control PBSs such as photonic band gaps, exposure of the PPC to an external magnetic field is a new approach besides adjusting the plasma parameters to control the properties of the PPC. It is well known that there are two famous magneto-optical effects: the Faraday effect

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and the Voigt effect, when a one dimensional PPC is exposed to an external magnetic field. In this study we study how the PBSs of a one dimensional PPC are tuned after being exposed to an external magnetic field. The tunable one dimensional PPCs consist of a plasma layer and a background material layer stacked alternately, and their behavior under the influence of the Faraday effect and the Voigt effect is studied in detail in the whole frequency ranges.

The transfer matrix method [19,20] is used here to calculate the PBSs of the one dimensional PPCs consisting of a plasma layer and a background material layer (air) stacked alternately. The present paper is organized as follows. The model and corresponding analytical formulas are first presented in section 2. Then the tunable PPCs based on both the Faraday effect and the Voigt effect are studied and discussed. Finally, conclusions are given in section 3.

2 Two magnetic field effects

To study the propagation of EM waves through the periodic structure, we assume that the background material layer is non-magnetic, and select a particular x -axis passing through the material along a direction normal to the layers. An external magnetic field B_0 is perpendicular or parallel to wave vector k . The dielectric function of the plasma layer and background material layer are ε and ε_b with thickness a and b , respectively, and $\Lambda = a + b$ is the period. When B_0 is applied to the PPC, the dielectric function of plasma layer can be expressed as follows [21]

$$\hat{\varepsilon} = 1 - \frac{\omega_p^2}{\omega^2} \left\{ \left[1 - j \frac{\nu}{\omega} - \frac{\omega_{ce}^2 \sin^2 \theta}{2 \left(1 - \frac{\omega_p^2}{\omega^2} - j \frac{\nu}{\omega} \right)} \right] \pm \left[\frac{\omega_{ce}^4 \sin^4 \theta}{4 \left(1 - \frac{\omega_p^2}{\omega^2} - j \frac{\nu}{\omega} \right)^2} + \frac{\omega_{ce}^2 \cos^2 \theta}{\omega^2} \right]^{1/2} \right\}^{-1}, \quad (1)$$

where ω is the frequency of incident waves, the damping factor ν denotes collisions in plasma layer, where $\omega_p = (n_e e^2 / \varepsilon_0 m)^{1/2}$ is plasma frequency, and n_e is the electron density of the plasma layer, e is absolute electron charge, m is electron mass, ε_0 is the permittivity in free space, $\omega_{ce} = eB_0/m$ is the cyclotron frequency of electrons, θ is the angle between wave vector number k and the direction of the external magnetic field B_0 .

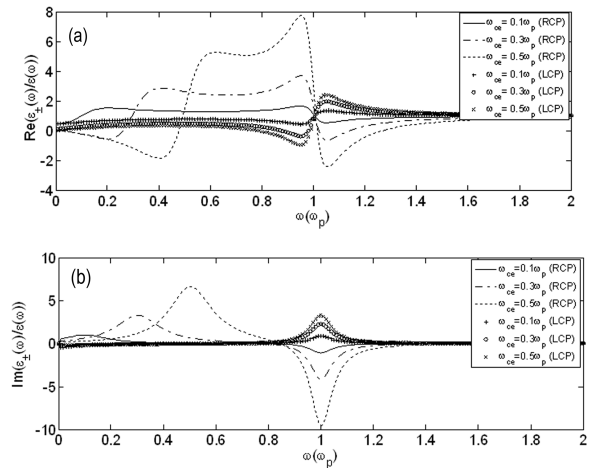
2.1 The Faraday effect ($\theta = 0$)

For the Faraday effect, the propagating direction of EM waves is along the external magnetic field direction, under the condition of $\theta = 0$. A rotation of the plane of polarization of EM waves is expected to arise from the difference between the dielectric constants of the left- and the right- handed circular polarizations. Then the different dielectric functions of the plasma layer can be

derived from Eq. (1) as $\theta = 0$, which are described as follows

$$\varepsilon_{\pm}(\omega) = 1 - \frac{\omega_p^2}{\omega[(\omega - j\nu) \mp \omega_{ce}]}, \quad (2)$$

where the subscripts $+$ and $-$ stand for right circular polarization (RCP) and left circular polarization (LCP), respectively. Fig. 1 shows the ratio of the complex dielectric constant $\varepsilon_{\pm}(\omega)$ to that without the external magnetic field $\varepsilon(\omega) = (1 - \omega_p^2/\omega(\omega - j\nu))$ versus the frequencies for different cyclotron frequencies, in which (a) depicts the real part of ε_{\pm} varying with the external magnetic field and (b) depicts the imaginary part of ε_{\pm} . It can be seen that ε_{\pm} is considerably altered by the external magnetic fields. Therefore, if one of the constituents of PC is the plasma layer, the PBSs are expected to be tunable since the complex dielectric constant varies with the external magnetic field. Specifically, the dielectric constants for RCP and LCP get different modulations in different frequency regions, especially in the range of $0 < \omega < \omega_p$, as shown in Fig. 1. In addition, just above the plasma frequency, ω_p , the modification of the complex dielectric constants by the external magnetic field is different for RCP and LCP. Well above plasma frequency ω_p , however, the modification is slight.



(a) Real part of ε_{\pm} , (b) Imaginary part of ε_{\pm}

Fig.1 Ratio of complex dielectric constant $\varepsilon_{\pm}(\omega)$ to that without the external magnetic field $\varepsilon(\omega)$ versus the frequencies for different cyclotron frequencies. (a) The real part of ε_{\pm} varying with the external magnetic field and (b) the imaginary part of ε_{\pm}

The band structures of one dimensional PPCs with a different external magnetic field and without an external magnetic field are given in Fig. 2. In Fig. 2(a)~(c) the real part of K is presented which indicates a normal PBSs and in Fig. 2(d)~(f) the imaginary part of K is presented which indicates an absorption PBSs. These figures show that when the external magnetic fields are applied, the PBSs of LCP and RCP behave differently. Without the external magnetic field, the PBSs do not depend on the polarization of the incident EM waves.

However, when the magnetic field is applied, the properties of PBSs are dependent on the polarization. Compared with those without an external magnetic field, for EM waves with LCP, both the normal and the absorption PBSs are shifted downwards. For EM waves with RCP, both the normal and absorption PBSs are shifted upwards. When the frequency of incident EM waves is well above the plasma frequency, it can be seen that the difference of normal PBSs is small under the conditions of with or without an applied external magnetic field. So is the difference of absorption PBSs. That is due to the weak modulation degree of the external magnetic field when the frequency of incident EM waves is well above the plasma frequency. In some frequency ranges below the plasma frequency ω_p , EM waves for LCP and RCP can be generated with PCs by applying an external magnetic field. Similarly, above the plasma frequency ω_p , as the PBSs for LCP and RCP are modulated differently by the external magnetic field, the photonic band gap window for LCP or RCP can be used to generate EM waves with RCP or LCP. Therefore, such a system can be used as a circular polarizer as studied in Ref. [22].

2.2 The Voigt effect ($\theta = \pi/2$)

For the Voigt effect, the propagating direction of EM waves is perpendicular to the external magnetic field direction, under the conditions of $\theta = \pi/2$. Then the

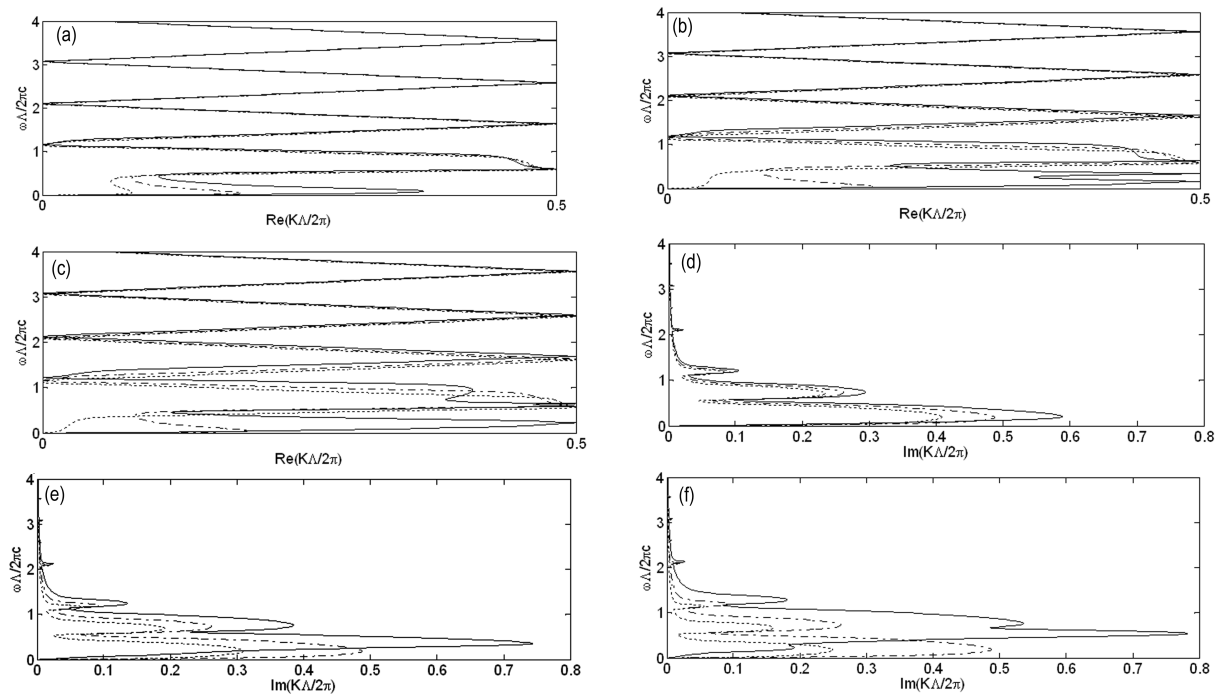
different dielectric functions of the plasma layer can be derived from Eq. (1) for $\theta = \pi/2$, which are described as follows

$$\varepsilon_{\parallel}(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega - j\nu)}, \quad (3)$$

and

$$\varepsilon_{\perp}(\omega) = 1 - \frac{\omega_p^2/\omega^2}{\left[1 - j\frac{\nu}{\omega} - \frac{\omega_{ce}^2/\omega^2}{(1 - \omega_p^2/\omega^2 - j\nu/\omega)}\right]}. \quad (4)$$

where the subscripts \parallel and \perp stand respectively for where the electric field of the EM waves is parallel to the external magnetic field which is denoted by E polarization and where the electric field of the EM waves is perpendicular to the external magnetic field which is denoted by H polarization. Fig. 3 shows the ratio of complex dielectric constant $\varepsilon_{\perp}(\omega)$ to that without the external magnetic field $\varepsilon(\omega) = 1 - \omega_p^2/\omega(\omega - j\nu)$ versus the frequencies for different cyclotron frequencies, in which (a) depicts the real part of ε_{\perp} varying with the external magnetic field and (b) depicts the imaginary part of ε_{\perp} . In the same way as those for Faraday effect, the dielectric constants for E polarization and H polarization get different modulations in different frequency regions. These will influence the PBSs, both for normal and absorption PBSs.



(a) Normal photonic band structures, $\omega_{ce} = 0.1\omega_p$, (b) Normal photonic band structures, $\omega_{ce} = 0.3\omega_p$, (c) Normal photonic band structures, $\omega_{ce} = 0.5\omega_p$, (d) Absorption photonic band structures, $\omega_{ce} = 0.1\omega_p$, (e) Absorption photonic band structures, $\omega_{ce} = 0.3\omega_p$, (f) Absorption photonic band structures, $\omega_{ce} = 0.5\omega_p$

Fig.2 Band structures of one dimensional PPCs with different external magnetic fields and without an external magnetic field. Fig. 2(a)~(c) the real part of K is presented which indicates a normal PBSs and Fig. 2(d)~(f) the imaginary part of K is presented which indicates an absorption PBSs. Solid and dash-dotted lines stand for the PBSs of RCP and LCP, and dotted lines stand for the case without an external magnetic field. Other parameters are given as follows: dimensionless variable $\omega_p\Lambda/2\pi c = 1$, plasma filling factor $f = a/\Lambda = 0.4$, damping factor $\nu = 0.1\omega_p$

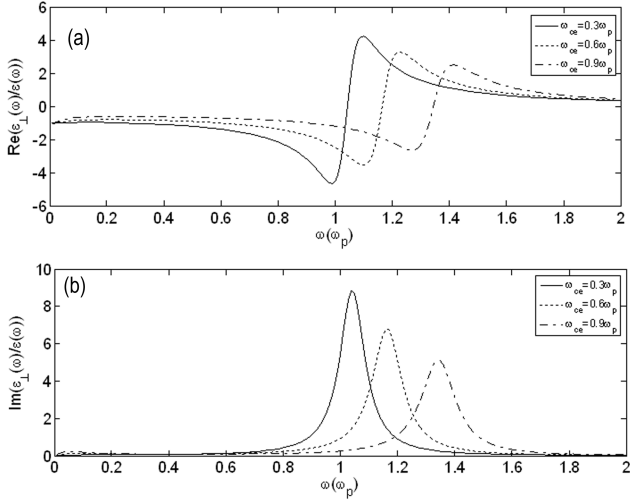
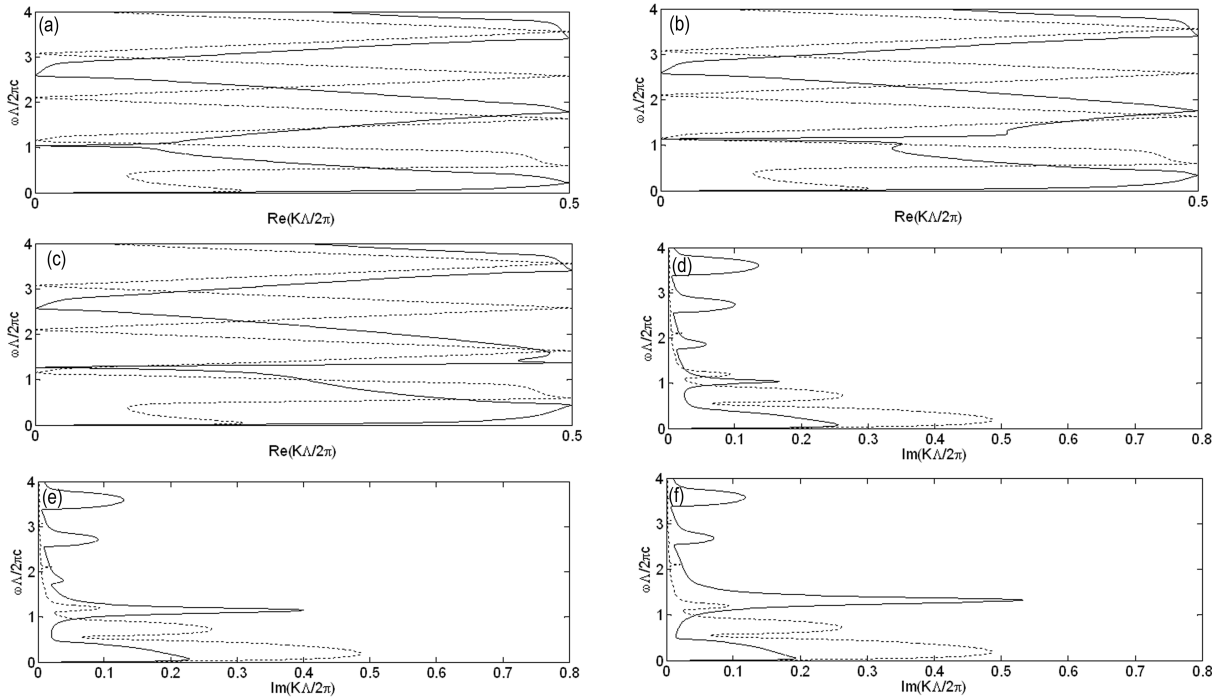


Fig.3 Ratio of complex dielectric constant $\varepsilon_{\perp}(\omega)$ to that without the external magnetic field $\varepsilon(\omega) = 1 - \omega_p^2/\omega(\omega - j\nu)$ versus the frequencies for different cyclotron frequencies, of which (a) depicts the real part of ε_{\perp} varying with an external magnetic field and (b) depicts the imaginary part of ε_{\perp}

Fig. 4 shows the band structures of E polarization and H polarization under different external magnetic fields. In Fig. 4(a)~(c) the real part of K is presented which indicates a normal PBSs and in Fig. 4(d)~(f) the imaginary part of K is presented which indicates an



(a) Normal photonic band structures, $\omega_{ce} = 0.3\omega_p$, (b) Normal photonic band structures, $\omega_{ce} = 0.6\omega_p$, (c) Normal photonic band structures, $\omega_{ce} = 0.9\omega_p$, (d) Absorption photonic band structures, $\omega_{ce} = 0.3\omega_p$, (e) Absorption photonic band structures, $\omega_{ce} = 0.6\omega_p$, (f) Absorption photonic band structures, $\omega_{ce} = 0.9\omega_p$

Fig.4 Band structures of one dimensional PPCs with different external magnetic fields and without an external magnetic field. In Fig. 4(a)~(c) the real part of K is presented which indicates a normal PBSs and in Fig. 4(d)~(f) the imaginary part of K is presented which indicates an absorption PBSs. Solid and dotted lines stand for the PBSs of E polarization and H polarization, respectively. The parameters are the same as used in Fig. 2

absorption PBSs. The PBSs for the E polarization are magnetic field independent, but for the H polarization they are magnetic field dependent. Below the plasma frequency ω_p , the difference of both normal and absorption PBSs is significant. Above and near the plasma frequency ω_p , the bands of H polarization are pushed upwards together with the bands of E polarization. Well above the plasma frequency ω_p , the modulation is much smaller which is similar to the situation for the Faraday effect. These results show that EM waves with E and H polarization can pass through or be blocked in some frequency ranges by switching the external magnetic field. Therefore, EM waves with fixed polarization direction can be selected under the influence of the Voigt effect. It can also be seen from Fig. 4(a)~(c) that there exists a strong dispersion when the external magnetic field is perpendicular to the electric field. For example, when $\omega_{ce} = 0.3\omega_p$, the resonance frequency point is near ω_p , when $\omega_{ce} = 0.6\omega_p$, the resonance frequency point is near $1.16\omega_p$, and when $\omega_{ce} = 0.9\omega_p$, the resonance frequency point is near $1.35\omega_p$. They are very consistent with the resonance frequency equation $\omega^2 = \omega_p^2 + \omega_{ce}^2$ [23].

3 Conclusions

In the present paper, we have studied exclusively how the normal and absorption PBSs of one dimensional PPCs are tuned after being exposed to an external mag-

netic field. Two special cases, Faraday effect and Voigt effect, are considered. Both the normal and absorption PBSs of one dimensional PPCs are calculated and discussed under the Faraday and the Voigt effects. The results show that due to the magnetic-optical effects, including the Faraday and Voigt effects, the PBSs of one dimensional PPCs are tuned because the dielectric constants of the micro plasma layer are modified differently in different frequency ranges, especially in the range below the plasma frequency ω_p .

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