Terahertz Plasma Waves in Two Dimensional Quantum Electron Gas with Electron Scattering*

ZHANG Liping (张丽萍)

School of Sciences, LanZhou University of Technology, Lanzhou 730050, China

Abstract We investigate the Terahertz (THz) plasma waves in a two-dimensional (2D) electron gas in a nanometer field effect transistor (FET) with quantum effects, the electron scattering, the thermal motion of electrons and electron exchange-correlation. We find that, while the electron scattering, the wave number along y direction and the electron exchange-correlation suppress the radiation power, but the thermal motion of electrons and the quantum effects can amplify the radiation power. The radiation frequency decreases with electron exchange-correlation contributions, but increases with quantum effects, the wave number along y direction and thermal motion of electrons. It is worth mentioning that the electron scattering has scarce influence on the radiation frequency. These properties could be of great help to the realization of practical THz plasma oscillations in nanometer FET.

Keywords: THz plasma waves, nanometer field effect transistor, electron scattering, radiation power, radiation frequency

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(Some figures may appear in colour only in the online journal)

1 Introduction

During the past decade, the study on terahertz (THz) plasmonic technologies and their applications in homeland security, biomedical imaging, radio astronomy, industrial controls, short range covert and space communications have recently been proceeding rapidly. The theoretical research for THz plasma waves indicates that the stable state can turn into unstable state with the appearance of high-frequency plasma waves, when a direct current gets through a field effect transistor (FET) with short-channel [1−5]. The basic reason of this instability is non-symmetrical boundary conditions. The plasma waves amplification owing to the device boundaries' wave reflection results in the generation of plasma waves, which lead to the electromagnetic radiation that its frequency is equal to the plasma waves frequency. The plasma waves frequency depends on dimensions of FET and when gate lengths reach a micron and sub-micron size the plasma waves frequency reaches the THz range. The generation of plasma waves in the FET channel can be also applied to the electromagnetic radiation's detection and the emission and the frequency mixing in the THz domain [6,7]. Moreover, the results of many experiments [8−15] show that the calculated characteristic plasma waves frequency corresponds roughly to the experimental emission and detection frequency, and when the drain current is greater than a certain well defined threshold value the emission appears, as predicted by the instability model. When the length of FET channel continuously decreases and reaches nanometer size, the quantum effects, such as quantum statistical pressures for electrons, the electron tunneling and many electron exchange correlation [16,17], become more and more important. These quantum effects may have an important influence on FET operations, for example, capacitance-voltage characteristics, current-voltage characteristics, and leakage current, which is due to the relevance of quantum confinement on the distribution of the channel charge. Thus, investigating the collective behavior of a quantum electron gas is absolutely essential to design a new generation of microelectronic devices, such as metallic nanometer structures, thin metal films, semiconductor quantum dots and quantum wells.

Electron scattering by impurities and/or phonons plays an important role on carrier transport in channel of FET, which is another factor that affects the condition of plasma waves instability. The external friction due to electron scattering results in the inhomogeneity of steady state. Theoretical investigations on the influence of electron scattering on the propagation characteristics of plasma waves in a ballistic FET show that the friction due to electron scattering by impurities and/or phonons results in a strongly inhomogeneous distributed potential in the channel and weakens the increment of the instability [18,19]. In fact, the gate length is much less than the gate width in a full tran-

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sistor channel in the standard experiment. In this case, the wave velocity and electron density not only rely on the coordinate x but also rely on the coordinate y. Therefore oblique plasma waves of which the wave vector in the y direction is not equal to zero should take part in the inductive signal between source and drain and amplify its longitudinal resonances. In such a geometry, the full channel of a FET becomes a wave guide with plasma waves' continuous spectrum. Under such conditions, the one dimensional model is out of place.

In this article, based on a quantum hydrodynamic model (QHM), the influence of the quantum effects and the electron scattering on the oblique plasma waves' instability in the two dimensional nanometer FET with asymmetrical boundary conditions is studied both analytically and numerically. The results show that the thermal motion of electrons and the quantum effects enhance the radiation power, but the friction due to electron scattering, the wave number along y direction and the electron exchange-correlation contributions reduce the radiation power. Furthermore, the radiation frequency increases with quantum effects, the wave number along y direction and thermal motion of electrons, but decreases with electron exchange-correlation contributions. It is worth mentioning that the electron scattering has scarce influence on the radiation frequency. The research conclusions are useful for device parameters' selection in experiment observing the instability of THz plasma waves. Therefore, besides evident assistance in plasma waves excitations in nanometer structures, studying the instability of THz plasma waves in nanometer FETs is necessary for potential applications.

2 Model

Electron-electron scattering rates can be large compared to other frequency scales of interest for sufficiently high electron concentrations. In this limit, the electrons behave as a fluid moving in the channel, and the system can be described by a hydrodynamic model. Moving away from this limit, some quantitative accuracy may be lost, but these approximations should still support qualitative analysis of the quantum effects of interest here. We choose a self-consistent QHM that was initially obtained [20,21] from the Wigner-Poisson system and this QHM has been widely used to study quantum plasmas and metallic nanostructures

$$\frac{\partial n}{\partial t}+\nabla\cdot(n\mathbf{v})=0, \tag{1}$$

$$\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}=-\frac{e}{m}\nabla\varphi+\frac{\hbar^2}{2m^2}\nabla\left(\frac{1}{\sqrt{n}}\nabla^2\sqrt{n}\right)-\frac{1}{m}\nabla V_{\mathrm{xc}}-\frac{1}{mn}\nabla P-\frac{\mathbf{v}}{\tau}, \tag{2}$$

where n is the electron density, $\mathbf{v}$ is the average electron flux velocity, m and e are the effective mass and the electron charge respectively, φ is the potential, P is pressure term related to the electron density n. Generally speaking, a polytropic relation is selected, $P=\bar{n}k_{\mathrm{B}}T_{\mathrm{e}}(n/\bar{n})^{\gamma}$, where k_{B} is the Boltzmann constant, $\bar{n}$ is a mean electron density, T_{e} is the electron temperature, and $\gamma=3$. The external friction associated with electron scattering via impurities and/or phonons introduces an supplementary term $\mathbf{v}/\tau$ in the right-hand side of Eq. (2), where τ is the momentum relaxation time.

The two dimensional free-carrier density in the channel where the channel width $\delta\to 0$ keeps a finite value, the potential is given [22],

$$en=C\varphi, \tag{3}$$

where C is the gate to channel capacitance per unit area. When the distance from gate to channel is less than the scope of the floating of the potential in the channel, this equation (gradual channel approximation equation) is valid. The second term of the right-hand in the Eq. (2), which is called Bohm potential, contains all the quantum effects of the system.

In general, V_{xc} is the exchange-correlation potential, an functional of the electron density n. The density distribution of an interaction electron gas in an external field can be obtained using an one-body Schrödinger-type equation containing an exchange-correlation potential. Such an exchange-correlation potential V_{xc} [23,24] is given by $V_{\mathrm{xc}}=-0.985\frac{e^2}{\varepsilon}n^{1/3}\{1+\frac{0.034}{a_{\mathrm{B}}^{*}n^{1/3}}\ln[1+18.376a_{\mathrm{B}}^{*}n^{1/3}]\}$, where $a_{\mathrm{B}}^{*}=\varepsilon\hbar^2/me^2$.

3 Plasma waves instability

To investigate the instability of this steady state, we put $n=n_0+n_1$, $v_x=v_0+v_{x1}$, $v_y=v_{y1}$ with $n_1,v_{x1},v_{y1}\sim\exp(-\mathrm{i}\omega t+\mathrm{i}kx+\mathrm{i}qy)$ (where k and q are the components of the wave vector along the x and y directions respectively), and linearize Eqs. (1)-(2) with respect to n_1, v_{x1} and v_{y1}. Then, we find

$$(\omega-kv_0)n_1-n_0(kv_{x1}+qv_{y1})=0, \tag{4}$$

$$(\omega-kv_0+\mathrm{i}\nu)v_{x1}=\Big[\frac{\hbar^2}{4m^2n_0}k^3+\frac{3k_{\mathrm{B}}T_{\mathrm{e}}}{n_0m}k-\Big(0.53\frac{e^2}{m\varepsilon}n_0^{-\frac{2}{3}}-3.77\frac{\hbar^2}{4m^2}n_0^{-\frac{1}{3}}\Big)k+\frac{e^2}{mC}k\Big]n_1, \tag{5}$$

$$(\omega-kv_0+\mathrm{i}\nu)v_{y1}=\Big[\frac{\hbar^2}{4m^2n_0}q^3+\frac{3k_{\mathrm{B}}T_{\mathrm{e}}}{mn_0}q-\Big(0.53\frac{e^2}{m\varepsilon}n_0^{-\frac{2}{3}}-3.77\frac{\hbar^2}{4m^2}n_0^{-\frac{1}{3}}\Big)q+\frac{e^2}{mC}q\Big]n_1. \tag{6}$$

We normalize electron flux velocity $\mathbf{v}$ and the thermal velocity of electron $v_{\mathrm{e}}=\sqrt{3k_{\mathrm{B}}T_{\mathrm{e}}/m}$ to the velocity of plasma waves $s=\sqrt{e^2n_0/mC}$, the length to L, the time to $\frac{L}{s}$, the frequency to $\frac{s}{L}$. This procedure gives

$$(\Omega-\beta K)\tilde{n}_1-K\tilde{v}_{x1}-Q\tilde{v}_{y1}=0, \tag{7}$$

$$(\Omega-\beta K+\mathrm{i}\gamma)\tilde{v}_{x1}-[HK^3+\alpha K]\tilde{n}_1=0, \tag{8}$$

$$(\Omega-\beta K+\mathrm{i}\gamma)\tilde{v}_{y1}-[HQ^3+\alpha Q]\tilde{n}_1=0, \tag{9}$$

where $\tilde{n}_1 = n_1/n_0$, $\tilde{v}_{x1} = v_{x1}/s$, $\tilde{v}_{y1} = v_{y1}/s$. Therefore, the dimensionless dispersion equation yields:

$$(\Omega - \beta K)(\Omega - \beta K + \mathrm{i}\gamma) - (HK^4 + \alpha K^2) - (HQ^4 + \alpha Q^2) = 0, \quad (10)$$

where $\alpha = 1 - E + F^2$. $\Omega(= \omega L/s)$, $K(= k/L)$ and $Q(= q/L)$ are the dimensionless frequency, the components of the dimensionless wave vector along the x and y axes, respectively. $\beta = \frac{v_0}{s}$ is dimensionless equilibrium velocity, $H = \frac{\hbar^2}{4m^2}\frac{1}{s^2 L^2}$ corresponding to quantum effects depends on the dimension of device. $E = 0.53\frac{e^2}{s^2 m\epsilon} n_0^{1/3} - 3.77\frac{\hbar^2}{s^2 m^2} n_0^{2/3}$ is electron exchange-correlation contributions. $F = \frac{v_e}{s}$ is dimensionless thermal velocity of electron depending on the temperature of electron gas. $\gamma = L/s\tau$ represents the external friction associated with electron scattering via impurities and/or phonons.

It can been seen from Eq. (10) that K has four roots. However, we find that K has only two real roots corresponding to waves propagating downstream and upstream by solving Eq. (10) using mathmatic. We set K_1 and K_2 respectively. We employ the asymmetrical boundary conditions [1], that is, the AC potential is equal to zero at the source and the current is equal to zero at the drain. Then, the boundary conditions is:

$$\tilde{n_1}(x = 0) = 0, \tilde{n}(L, t)\tilde{v}(L, t) = \beta, \quad (11)$$

where L is the length of channel of FET. By grounding the source either directly or via very large capacitance presenting a short at plasma waves frequency and by attaching the drain to the power supply via an inductance that presents an open circuit at plasma waves frequency these boundary conditions can be realized. From Eq. (11), we obtain:

$$\exp[\mathrm{i}(K_1 - K_2)] = \frac{\beta + \frac{HK_2^3 + \alpha K_2}{\Omega - \beta K_2 + \mathrm{i}\gamma}}{\beta + \frac{HK_1^3 + \alpha K_1}{\Omega - \beta K_1 + \mathrm{i}\gamma}}, \quad (12)$$

Eq. (12) allows us to find both the real and imaginary parts of the complex frequency $\Omega = \Omega^{'} + \mathrm{i}\Omega^{''}$ by separating real and imaginary part numerically, and the sign of the imaginary part will determine the stability of the steady state. When the imaginary part of $\Omega^{''}$ is larger than zero, the plasma waves are unstable.

When the size of device is a litter bigger, the quantum effects due to size may be neglected. That is, in this case, we put $H \sim 0$ in Eq. (10). Eq. (10) reduces to $(\Omega - \beta K)(\Omega - \beta K + \mathrm{i}\gamma) - \alpha(K^2 - Q^2) = 0$. For given Ω, we find two other values corresponding to oblique waves propagating downstream and upstream: $K_{1,2} = [-q \pm \sqrt{(q^2 - 4pr)}]/2p$, with $q = -2\beta\Omega - \mathrm{i}\gamma\beta$, $p = 2(\beta^2 - \alpha)$ and $r = \Omega^2 - \alpha Q^2 + \mathrm{i}\Omega\gamma$.

If the friction due to electron scattering is neglected, we can obtain an analytical solution for small drift velocities compared to the plasma waves velocity ($\beta \ll 1$). When β is equal to zero, the solution of Eq. (12) is $\Omega^2 = [(n^2\pi^2 H/2 + \alpha)^2 - \alpha^2]/4H(HQ^4 + \alpha Q^2)$ where $n = 1, 3, 5....$ It is clear that the plasma oscillation modes are modified by the electron exchange-correction contributions E and the thermal velocity of electrons F. The oscillation frequency is up-shifted by the thermal motion of electrons and down-shifted by the electron exchange-correction contributions. That is, the radiation frequency is reduced by the electron exchange-correction contributions and enhanced by the thermal motion of electrons.

Overall, the quantum effects, the oblique propagation of plasma waves, the external friction associated with electron scattering by impurities and/or phonons, the electron exchange-correction contributions and the thermal motion of electrons have influence on both the instability increment corresponding to the radiation power and the radiation frequency. In order to analyze the essential features of the plasma waves instability, we numerically solve Eq. (10) and (12). The steady flow is called unstable if $\Omega^{''} > 0$, i.e., waves grow. Fig. 1 indicates the instability increment and the radiation frequency as a function of β for different H, E and F. It can be seen from Fig. 1 that the instability range of the Mach number β expands with the quantum effects H and the thermal motion of electrons F, but shrinks with the electron exchange-correction contributions E, the instability increment ($\Omega^{''}$) and the radiation frequency ($\Omega^{'}$) increase with the quantum effects H and the thermal motion of electrons F but decrease with electron exchange-correction contributions E. In Fig. 2, we show the dependence of the instability increment and the radiation frequency on β for different wave number along y direction (Q) and external friction of electron scattering (γ). As shown in Fig. 2, when the oblique propagation of plasma waves and the electron scattering are considered the instability increment and the instability range of the Mach number

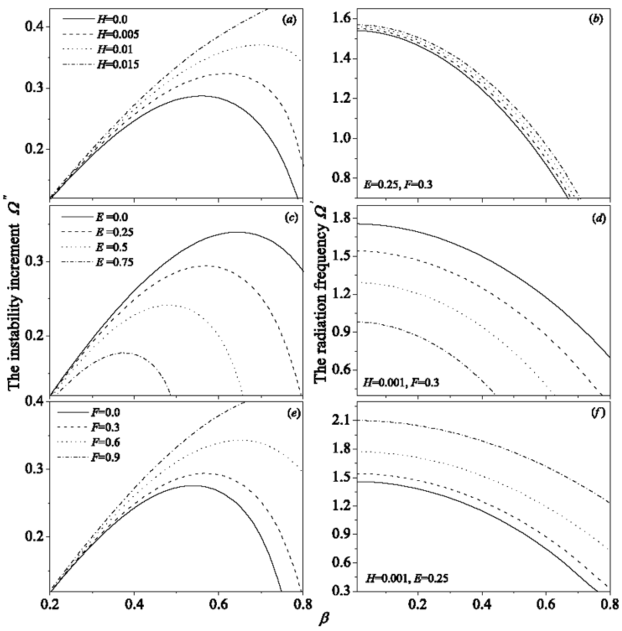

Fig.1 The instability increment Ω'' and the radiation frequency Ω' as a function of β for different H, E and F with $\gamma = 0.1$ and Q=0.6

β decrease, but the radiation frequency increases. Fig. 2(d) shows that the influence of the friction of electron scattering on the radiation frequency is much weak. Furthermore, it can be seen from Figs. 1 and 2 that for low β, the instability increment varies smoothly with the quantum effects, the exchange-correction contributions, the wave number along y direction and the thermal motion of electrons, but for high β, the instability increment varies with the quantum effects, the exchange-correction contributions, the wave number along y direction and the thermal motion of electrons significantly. However, from Fig. 2(c), one can see that the influence of the friction of electron scattering on the instability increment is obvious for low β.

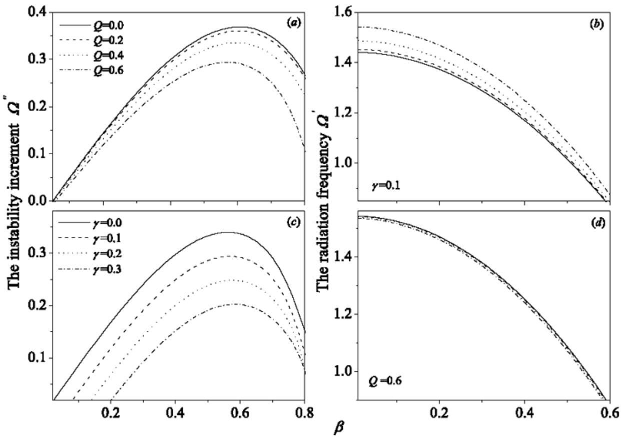

Fig.2 The instability increment Ω'' and the radiation frequency Ω' as a function of β for different Q and γ with $H = 0.001$, $E = 0.25$ and F=0.3

In Figs. 3 and 4, we show the instability increment and the radiation frequency as a function of the thermal motion of electrons (F) for different H, E, Q and γ. For general case, the instability increment and the radiation frequency increase more slowly for a little weaker thermal motion of electrons, however, the instability increment and the frequency increase quickly for a little stronger thermal motion of electrons. As shown in Fig. 3, when the thermal motion of electron is weaker, the quantum effects and the exchange-correction contributions have significant influence on the instability increment and the frequency increases, but when the thermal motion of electron is stronger, the quantum effects and the exchange-correction contributions have weak influence on the instability increment and the radiation frequency.

Figs. 5 and 6 plot the instability increment and the radiation frequency against the wave number along y direction (Q) for different H, E, F and γ. From Figs. 5 and 6, we can find that, when the wave number along y direction is small, the instability increment decreases and the radiation frequency increases slowly with the wave number along y direction, but when the wave number along y direction is large, the instability increment decreases and the radiation frequency increases sharply with the wave number along y direction. The current-carrying state become unstable in the transistor channel against spontaneous generation of plasma waves in a nanometer EFT with asymmetrical source and drain boundary conditions. A much higher-quality factor ($\omega\tau$) is related with much higher frequency. When the size of device becomes smaller and smaller the quantum effects become more and more important. So that, when the size of device decreases the radiation frequency and the radiation power increase.

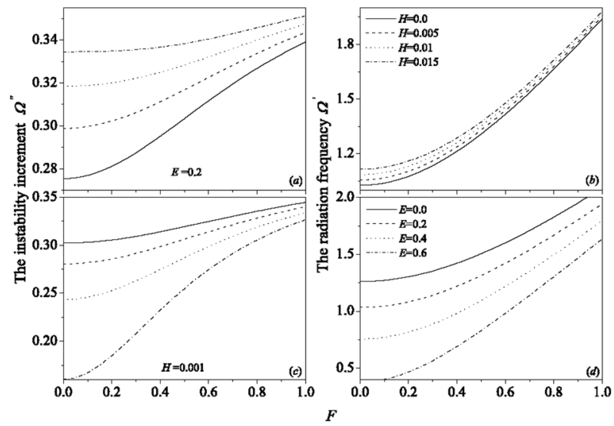

Fig.3 The instability increment Ω'' and the radiation frequency Ω' as a function of F for different H and C with Q=0.6, β=0.5 and γ=0.1

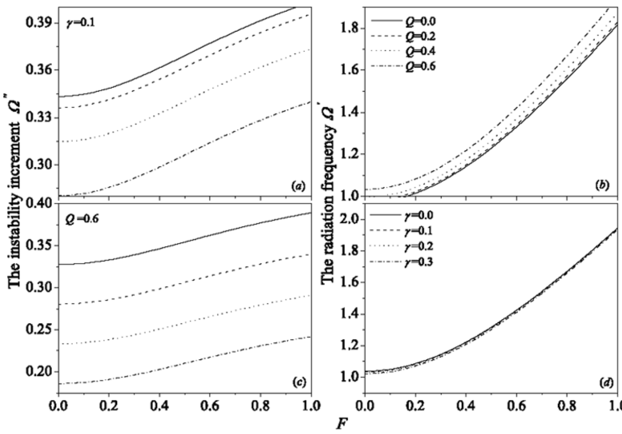

Fig.4 The instability increment Ω'' and the radiation frequency Ω' as a function of F for different Q and γ with H=0.001, β=0.5 and E=0.2

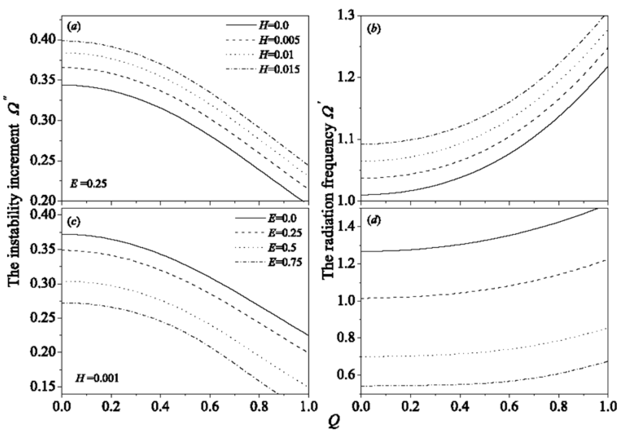

Fig.5 The instability increment Ω'' and the radiation frequency Ω' as a function of Q for different H and C with F=0.3, β=0.5 and γ=0.1

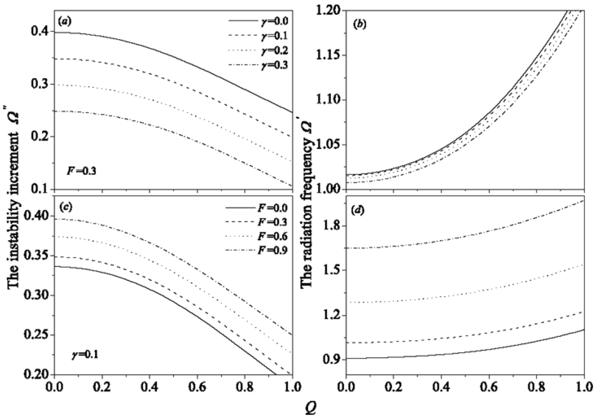

Fig.6 The instability increment Ω'' and the radiation frequency Ω' as a function of Q for different γ and F with H=0.001, β=0.5 and E=0.25

4 Conclusions

In conclusion, the current instability and resulting plasma waves generation due to the waves' reflection from the device boundaries are studied by the quantum hydrodynamic model. We extended the analysis to the pure two dimensional quantum electron gas realizable in a short FET channel with external friction due to electron scattering. We found that the quantum effects and the thermal motion of electrons enhanced the radiation power, but the electron exchange-correlation contributions, the oblique propagation of plasma waves and the friction due to electron scattering reduced the radiation power. The quantum effects, the oblique propagation of plasma waves and the thermal motion of electrons enhanced the radiation frequencies. It is worth mentioning that the electron scattering has scarce influence on the radiation frequency. These properties could be of great help to the realization of practical THz plasma oscillations in nanometer FET and to the choice of device parameters in the experiment observing the instability of THz plasma waves.

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E-mail address of ZHANG Liping: zhanglp@lut.cn