Dust Acoustic Compressive Waves in a Warm Dusty Plasma Having Non-Thermal Ions and Non-Isothermal Electrons

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Abstract In this article an investigation is presented on the properties of dust acoustic (DA) compressive solitary wave propagation in an adiabatic dusty plasma, including the effect of non-thermal positive and negative ions and non-isothermal electrons. The reductive perturbation method has been employed to derive the lower degree modified Kadomtsev-Petviashivili (mK-P), 3D Schamel-Korteweg-de-Vries equation or modified Kadomtsev-Petviashivili (mK-P) equations for dust acoustic solitary waves in a homogeneous, unmagnetized and collisionless plasma whose constituents are non-isothermal electrons, singly charged positive and negative non-thermal ions and massive charged dust particles. The stationary analytical solutions of the lower degree mK-P and mK-P equations are numerically analyzed, where the effect of various dusty plasma constituents on DA solitary wave propagation is taken into account. It is observed that both the ions in dusty plasma play a key role in the formation of DA compressive solitary waves, and also the ion concentration and non-isothermal electrons control the transformation of the compressive potentials of the waves.

Keywords: dusty plasma, reductive perturbation method, non-isothermal ions, compressive solitary wave

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(Some figures may appear in colour only in the online journal)

1 Introduction

Wave and instabilities in low temperature dusty plasmas have been widely studied in the last few decades, since the presence of extremely massive charged dust particles plays an imperative role in understanding the electrostatic disturbances in space plasma environments as well as in laboratory plasma devices [1]. The presence of dust grains in a two component electron-ion plasma is responsible for the appearance of new types of electrostatic waves, including solitary or shock waves, and have been reported by many researchers in both theoretical and experimental points of view [1−11]. One of these electrostatic waves is the low frequency dust-acoustic (DA) mode in an unmagnetized dusty plasma whose constituents are charged dust fluid and Boltzmann distributed electrons and ions. It was Rao et al. [12] who were the first to report theoretically the existence of these DA solitary waves (SWs). They showed the formation of a rarefactive type of DASWs solution in dusty plasmas, and the predictions of DA solitary waves were conclusively verified by laboratory experiments [13]. In continuation with this, Mamun et al. [14] investigated the nonlinear DA waves in a two component unmagnetized dusty plasma consisting of a negatively charged cold dust fluid and isothermal electrons. Lin et al. [15] considered the non-thermal ions in a dusty plasma to derive a Korteweg-de Vries (KdV) equation for DA waves, and it was found that the non-thermal ions have a very important effect on the propagation of DA solitary waves. In another work of Mamun [16] it was reported that the adiabatic effect of inertialess electron and ion fluids has significantly modified the basic properties of DA solitary waves. In continuation of these, a number of theoretical investigations [17−20] have been made on DASWs by assuming a three components unmagnetized dusty plasma consisting of a negatively charged cold dust fluid and inertialess isothermal electron and ion fluids. These works are valid only for a cold dust fluid with isothermal electrons and ions.

It has also been confirmed from both theoretical and experimental observations that the presence of negative ions in dusty plasma plays an important role in many aspects, including the charging of the dust particles. In the experimental work of Adhikary et al. [10] the formation of rarefactive DIAWs under influence of negative ions in a dusty plasma was presented together with their characteristic properties. In another study of Roy et al. [21] on the role of negative ions with dust charge fluctuation, by estimating the dispersion relation for a DA wave it was shown that the low temperature negative ions can reduce both frequency and damping of the dust acoustic waves.

Some numerical investigations on linear and nonlinear DAW showed a significant amount of ions being trapped in the wave potential, which implies that there is a departure from Boltzmann ion distribution and one would encounter vortex-like ion distribution in phase space [22−25]. On the other hand, the nonlinear behavior of electrostatic waves in a plasma with this non-isothermal state [26−29] has received considerable attention and has been studied by a number of authors in the last few years in the context of unmagnetized and magnetized plasmas [30,31]. Mamun et al. [24] investigated the effects of vortex-like and non-thermal ion distributions within the small amplitude regime by using the modified K-dV equation and they concluded the possibility of the coexistence of large amplitude rarefactive as well as compressive dust-acoustic solitary waves, whereas these structures appear independently when the wave amplitudes become infinitely small. Dorranian and Sabetkar [32] reported that by increasing non-thermal ion population, the amplitude of the solitary wave decreases, while the width of solitary waves is increased. Recently Dev et al. [33] also studied the effect of non-thermal electrons and vortex-like electron distributions in compressive, rarefactive solitary waves and spiky solitary waves in a clod dusty plasma by deriving the K-P equation and mK-P equations.

Very recently Adhikary et al. derived [34] the mK-dV equation in a warm dusty plasma containing non-isothermal electrons and non-thermal positive and negative ions. So far as it is concerned, the solitary wave in an unmagnetized warm dusty plasma containing non-isothermal electrons and non-thermal positive and negative ions has not yet been studied in detailed 3D form of mK-dV and Schamel-K-dV equation. In this paper, a detailed investigation of the propagation characteristics of a dust acoustic wave in an unmagnetized warm dusty plasma containing non-isothermal electrons and non-thermal positive and negative ions is reported with derivation of the lower degree modified K-P, 3D form of Schamel-K-dV or mK-P equation. Here the effects of non-thermal ions as well as the non-isothermal electrons are taken into account, and their thermal effects are considered in an analytical treatment for the small amplitude compressive solitary wave limit.

2 Basic and normalized equations

In the present plasma model we consider an electrostatic dust acoustic solitary wave with extremely low phase velocity which is followed by the negatively charged massive dust particles. Here the pressure provides the restoring force and the inertia comes from the dust mass. The dynamics of the dust particles in a one dimensional dust acoustic wave in such a dusty plasma system can be described by the following basic equations

$$\frac{\partial n_{\rm d}}{\partial t} + \nabla.\left(n_{\rm d}\bar{v}_{\rm d}\right) = 0, \tag{1}$$

$$\left(\frac{\partial}{\partial t} + \bar{v}_{\rm d}\cdot\nabla\right)\bar{v}_{\rm d} + \frac{1}{m_{\rm d}n_{\rm d}}\nabla p_{\rm d} = -\frac{q_{\rm d}}{m_{\rm d}}\nabla\varphi, \tag{2}$$

$$\left(\frac{\partial}{\partial t} + \bar{v}_{\rm d}\cdot\nabla\right)p_{\rm d} + \gamma p_{\rm d}\nabla\cdot\bar{v}_{\rm d} = 0, \tag{3}$$

$$\nabla^2\varphi = 4\pi e\left(n_{\rm e} + n_{\rm n} - n_{\rm p} + Z_{\rm d}n_{\rm d}\right). \tag{4}$$

The non-thermal number density of positive ion $n_{\rm p}$, negative ion $n_{\rm n}$ can be described by the following relations,

$$n_{\rm p} = n_{\rm p0}\left(1 + \alpha\phi + \alpha\phi^2\right)\exp\left(-z_{\rm p}\phi\right), \tag{5}$$

$$n_{\rm n} = n_{\rm n0}\left\{1 + \alpha\sigma_{\rm p}\phi + \alpha\left(\sigma_{\rm p}\phi\right)^2\right\}\exp\left(z_{\rm n}\sigma_{\rm p}\phi\right), \tag{6}$$

with $\alpha = 4\gamma_1/1+3\gamma_1$ and $\sigma_{\rm p} = T_{\rm p}/T_{\rm n}$, where $n_{\rm d}$ is the number density of the negatively charged stationary dust particles in plasma, $Z_{\rm d}$ is the number of electrons residing on the dust surface at equilibrium, $p_{\rm d}$ is the pressure of the dust fluid, e is the electron charge, $m_{\rm p}$ ($m_{\rm n}$) is positive (negative) ion mass, $z_{\rm p}$ ($z_{\rm n}$) is positive (negative) ion charge state, $v_{\rm d}$ is dust fluid velocity, $T_{\rm e}$ ($T_{\rm p}$) is electron (positive ion) temperature, $\kappa_{\rm B}$ is Boltzmann constant, φ is electrostatic potential, γ_1 is the population of non-thermal ions in the plasma. The adiabatic index $\gamma = 5/3$ [$=(2+D)/D$, D is the number of degrees of freedom] is due to the three dimensional geometry of the system.

The non-isothermal electrons $n_{\rm e}$ can be described by the following relations

$$n_{\rm e} = n_{\rm e0}\left\{1 + (\beta\phi) - b\,(\beta\phi)^{3/2} + \frac{1}{2}(\beta\phi)^2 - ....\right\}, \tag{7}$$

with $\beta = T_{\rm p}/T_{\rm e}$, $b = 4(1-\gamma_2)/3\sqrt{\pi} > 0$, where the parameter γ_2 is defined as $\gamma_2 = T_{\rm ef}/T_{\rm et}$, in which $T_{\rm ef}$ and $T_{\rm et}$ are temperatures of free electrons and trapped electrons in the plasma, respectively. The parameter γ_2 determines the nature of the distribution function, giving plateau if $\gamma_2 = 0$ and a dip if $\gamma_2 < 0$, and a hump shape formed if $\gamma_2 > 0$. However, γ_2=1 corresponds to the Maxwellian distribution of the electrons. In the present plasma system the range of γ_2 will be considered as $0 < \gamma_2 < 1$ for the non-isothermal electrons.

Now, considering $N_{\rm d}$ dust number density normalized by its equilibrium value $n_{\rm d0}$, $V_{\rm d}$ is the dust-fluid velocity normalized by $c_{\rm sd} = \left(Z_{\rm d}\kappa_{\rm B}T_{\rm p}/m_{\rm d}\right)^{1/2}$, ϕ is the DA wave potential normalized by $\kappa_{\rm B}T_{\rm p}\varphi/e$, the time variable T is normalized by $\omega_{\rm pd}^{-1} = \left(m_{\rm d}/4\pi n_{\rm d0}Z_{\rm d}^2e^2\right)^{1/2}$, the space variable X is normalized by $\lambda_{\rm Dd}^{-1} = \left(4\pi n_{\rm d0}Z_{\rm d}e^2/\kappa_{\rm B}T_{\rm p}\right)^{1/2}$, pressure $p_{\rm d}$ is normalized by $P_{\rm d} = n_{\rm d0}\kappa_{\rm B}T_{\rm d}$ and from Eqs. (1)-(4) we get

$$\left(\frac{\partial}{\partial T} + \nabla\cdot\bar{V}_{\rm d}\right)N_{\rm d} = 0, \tag{8}$$

$$\left(\frac{\partial}{\partial T}+\bar{V}_{\mathrm{d}}\nabla\right)\bar{V}_{\mathrm{d}}+\frac{\sigma_{\mathrm{d}}}{N_{\mathrm{d}}}\nabla P_{\mathrm{d}}=\nabla\phi, \tag{9}$$

$$\left(\frac{\partial}{\partial T}+\bar{V}_{\mathrm{d}}\cdot\nabla\right)P_{\mathrm{d}}+\frac{5}{3}P_{\mathrm{d}}\nabla\cdot\bar{V}_{\mathrm{d}}=0, \tag{10}$$

$$\nabla^2\phi=p_1\phi-p_2\phi^{3/2}+p_3\phi^2+(N_{\mathrm{d}}-1), \tag{11}$$

with the overall charge neutrality condition

$$n_{\mathrm{e0}}=n_{\mathrm{p0}}-Z_{\mathrm{d}}n_{\mathrm{d0}}-n_{\mathrm{n0}}. \tag{12}$$

And $\mu_{\mathrm{p}}=n_{\mathrm{p0}}/Z_{\mathrm{d}}n_{\mathrm{d0}}$, $\sigma_{\mathrm{d}}=T_{\mathrm{d}}/Z_{\mathrm{d}}T_{\mathrm{p}}$, $\mu_{\mathrm{n}}=n_{\mathrm{n0}}/Z_{\mathrm{d}}n_{\mathrm{d0}}$, $n_{\mathrm{e0}}/Z_{\mathrm{d}}n_{\mathrm{d0}}=\mu_{\mathrm{e}}$, $p_1=\left\{\mu_{\mathrm{e}}\beta+\mu_{\mathrm{n}}(z_{\mathrm{n}}+\alpha\sigma_{\mathrm{p}})-\mu_{\mathrm{p}}(\alpha-z_{\mathrm{p}})\right\}$, $p_2=b\mu_{\mathrm{e}}\beta^{3/2}$, $p_3=\beta\mu_{\mathrm{e}}/2+\mu_{\mathrm{n}}(z_{\mathrm{n}}^2/2+\alpha\sigma_{\mathrm{p}}z_{\mathrm{n}}+\alpha(\sigma_{\mathrm{p}})^2)-\mu_{\mathrm{p}}(z_{\mathrm{p}}^2/2-\alpha z_{\mathrm{p}}+\alpha)$ are considered.

3 Derivation of lower degree modified K-P equation

Now, to derive the lower degree mK-P equation for the propagation of small but finite amplitude DASW, we use the standard reductive perturbation technique in which the independent variables ξ and τ are stretched as $\xi=\varepsilon^{1/4}(X-V_0T)$, $\eta=\varepsilon^{1/2}Y$, $\zeta=\varepsilon^{1/2}Z$ and $\tau=\varepsilon^{3/4}T$, V_0 is the phase speed (normalized by c_{sd}) of the wave along the x-direction and ε is a small nonzero constant measuring the weakness of the dispersion. The dependent variables $N_{\mathrm{d}}, V_{\mathrm{d}}, P_{\mathrm{d}}$ and ϕ can be expanded in power series of ε as

$$N_{\mathrm{d}}=1+\varepsilon N_{\mathrm{d}}^{(1)}+\varepsilon^{3/2}N_{\mathrm{d}}^{(2)}+....., \tag{13}$$

$$V_{\mathrm{d}x}=\varepsilon V_{\mathrm{d}x}^{(1)}+\varepsilon^{3/2}V_{\mathrm{d}x}^{(2)}+....., \tag{14}$$

$$P_{\mathrm{d}}=1+\varepsilon P_{\mathrm{d}}^{(1)}+\varepsilon^{3/2}P_{\mathrm{d}}^{(2)}+....., \tag{15}$$

$$\phi=\varepsilon\phi^{(1)}+\varepsilon^{3/2}\phi^{(2)}+\varepsilon^2\phi^{(3)}...., \tag{16}$$

$$V_{\mathrm{d}y,z}=\varepsilon^{5/4}V_{\mathrm{d}y,z}^{(1)}+\varepsilon^{7/4}V_{\mathrm{d}y,z}^{(2)}+.... . \tag{17}$$

Substituting the stretched co-ordinates and the expressions for $N_{\mathrm{d}}, V_{\mathrm{d}}, P_{\mathrm{d}}$ and ϕ into the normalized basic equations Eqs. (8)-(11) and equating the coefficients of the lowest degree of ε, i. e., $\left(\varepsilon^{5/4}\right)$ we get,

$$V_{\mathrm{d}}^{(1)}=\frac{3V_0\phi^{(1)}}{\left(5\sigma_{\mathrm{d}}-3V_0^2\right)}, \tag{18}$$

$$N_{\mathrm{d}}^{(1)}=\frac{3\phi^{(1)}}{\left(5\sigma_{\mathrm{d}}-3V_0^2\right)}, \tag{19}$$

$$P^{(1)}=\frac{5\phi^{(1)}}{\left(5\sigma_{\mathrm{d}}-3V_0^2\right)}. \tag{20}$$

Together with dispersion relation,
$V_0=$

$$\left[\frac{5}{3}\sigma_{\mathrm{d}}+\frac{1}{\{(\mu_{\mathrm{p}}-\mu_{\mathrm{n}}-1)\beta+\mu_{\mathrm{n}}(z_{\mathrm{n}}+\alpha\sigma)-\mu_{\mathrm{p}}(\alpha-z_{\mathrm{p}})\}}\right]^{1/2}. \tag{21}$$

From the next higher-order of ε, i.e., $\left(\varepsilon^{7/4}\right)$,

$$\frac{\partial N_{\mathrm{d}}^{(1)}}{\partial\tau}-V_0\frac{\partial N_{\mathrm{d}}^{(2)}}{\partial\xi}+\frac{\partial V_{\mathrm{d}}^{(2)}}{\partial\xi}+\left(\frac{\partial V_{\mathrm{d}}^{(1)}}{\partial\eta}+\frac{\partial V_{\mathrm{d}}^{(1)}}{\partial\zeta}\right)=0, \tag{22}$$

$$\frac{\partial V_{\mathrm{d}}^{(1)}}{\partial\tau}-V_0\frac{\partial V_{\mathrm{d}}^{(2)}}{\partial\xi}-\frac{\partial\phi^{(2)}}{\partial\xi}+\sigma_{\mathrm{d}}\frac{\partial P^{(2)}}{\partial\xi}=0, \tag{23}$$

$$\frac{\partial P^{(1)}}{\partial\tau}-V_0\frac{\partial P^{(2)}}{\partial\xi}+\frac{5}{3}\frac{\partial V_{\mathrm{d}}^{(2)}}{\partial\xi}+\frac{5}{3}\left(\frac{\partial V_{\mathrm{d}}^{(1)}}{\partial\eta}+\frac{\partial V_{\mathrm{d}}^{(1)}}{\partial\zeta}\right)=0, \tag{24}$$

$$\frac{\partial}{\partial\xi}\left(\frac{\partial V_{\mathrm{d}}^{(1)}}{\partial\eta}+\frac{\partial V_{\mathrm{d}}^{(1)}}{\partial\zeta}\right)=\frac{3V_0}{(5\sigma_{\mathrm{d}}-3V_0^2)}\left(\frac{\partial^2\phi^{(1)}}{\partial\eta^2}+\frac{\partial^2\phi^{(1)}}{\partial\zeta^2}\right), \tag{25}$$

$$\frac{\partial^3\phi^{(1)}}{\partial\xi^3}=p_1\frac{\partial\phi^{(2)}}{\partial\xi}-p_2\frac{3}{2}\frac{\partial}{\partial\xi}\left(\phi^{(1)}\right)^{1/2}+\frac{\partial}{\partial\xi}N_{\mathrm{d}}^{(2)}. \tag{26}$$

Substituting the values from Eqs. (18) to (21) into Eqs. (22) to (26), followed by a straight forward elimination of $N_{\mathrm{d}}^{(2)}$, $V_{\mathrm{d}}^{(2)}$, $P^{(2)}$, $\phi^{(2)}$, finally we obtain the following lower degree mK-P equation

$$\frac{\partial}{\partial\xi}\left(\frac{\partial\phi^{(1)}}{\partial\tau}+A_1\left(\phi^{(1)}\right)^{1/2}\frac{\partial}{\partial\xi}\phi^{(1)}+B\frac{\partial^3\phi^{(1)}}{\partial\xi^3}\right)+C\left(\frac{\partial^2\phi^{(1)}}{\partial\eta^2}+\frac{\partial^2\phi^{(1)}}{\partial\zeta^2}\right)=0, \tag{27}$$

where, the nonlinear coefficient A, the dispersion coefficient B and the transverse coefficient C are given by

$$A_1=\frac{p_2}{p_1}\frac{\left(3V_0^2-5\sigma_{\mathrm{d}}\right)}{4V_0},\quad B=\frac{\left(3V_0^2-5\sigma_{\mathrm{d}}\right)}{6p_1V_0},\quad C=\frac{V_0}{2}.$$

Eq. (27) represents the lower degree mK-P equation which describes the nonlinear propagation of the DA waves in an electronegative dusty plasma with non-thermal ions and non-isothermal electrons. To find its solution we use the transformation $\chi=(l\xi+m\eta+n\zeta-U\tau)$ and $\phi^{(1)}(\xi,\eta,\zeta,\tau)=\psi(\chi)$, then the above equation becomes

$$Bc^2l^4\frac{\mathrm{d}^2\psi}{\mathrm{d}\chi^2}+\left[C\left(m^2+n^2\right)-Ul\right]\psi+\frac{2A_1l^2}{3}\psi^{\frac{3}{2}}=0. \tag{28}$$

To derive the required solution of the lower degree modified K-P Eq. (27) we used the well-known tanh-method and the transformation $z=\tanh(\chi)$ and $\psi(\chi)=W(z)$, then Eq. (28) becomes

$$\frac{2A_1}{3}W^{3/2}l^2+Bc^2l^4\left(1-x^2\right)^2\frac{\mathrm{d}^2W}{\mathrm{d}x^2}-Bc^2l^4x\left(1-x^2\right)\frac{\mathrm{d}W}{\mathrm{d}x}+\left\{C\left(m^2+n^2\right)-Ul\right\}W=0. \tag{29}$$

For finding the series solution of Eq. (29), substituting $W(z)=\sum_{r=0}^{\infty}a_rz^{\rho+r}$ and using leading order analysis of finite terms gives $r=4$ and $\rho=0$, and then $W(z)$ becomes $W(z)=\left(a_0+a_1z+a_2z^2+a_3z^3+a_4z^4\right)=\left\{a_0\left(1-z^2\right)\right\}^2$. Now by referring to the value of $W(z)$, the required stationary solution of the lower degree mK-P equation is

$$\phi^{(1)} = \phi_{m1}\,\mathrm{sech}^4\left(\frac{\chi}{\omega_1}\right), \tag{30}$$

where $\phi_{m1} = \dfrac{225}{64}\Big[\{U\,l + C\,(m^2+n^2)\}/A_1\,l^2\Big]^2$ and $\omega_1 = 4\Big[Bl^3/\{U\,l + C\,(m^2+n^2)\}\Big]^{\frac{1}{2}}$ are the amplitude and width of the solitary waves, respectively, and l, m, n represent the direction cosines of the angle made by the propagation direction with the x-axis, y-axis and z-axis and U is the velocity.

The non-isothermal effect enhanced the amplitude of the lower degree mK-P equation solution but not its width. Since ϕ_{m1} is always positive, therefore the solution of the lower degree mK-P equation admits compressive solitions only.

4 Derivation of modified K-P (3D form of Schamel-K-DV) equation

For some particular plasma parameters, particularly for Maxwellian electrons if $A_1 = 0$ i.e. $b = 0$, then the amplitude of the lower degree mK-P equation $\phi_{m1} \to \infty$ and the lower degree mK-P equation does not offer a soliton solution. The lower degree mK-P equation is, therefore, inadequate, and we have to find another equation in a different degree to study the nonlinear properties of the DA waves. So for further amplification we familiarize with $b = \varepsilon^{1/2} b_1$ i.e. $b_1 \neq 0$ (Schamel [26−29]). The non-isothermal electrons n_{e} can be described by the following relations

$$n_{\mathrm{e}} = n_{\mathrm{e0}}\left\{1 + (\beta\phi) - \varepsilon^{\frac{1}{2}} b_1 (\beta\phi)^{3/2} + \frac{1}{2}(\beta\phi)^2 - ....\right\}. \tag{31}$$

The normalized Eq. (11) becomes

$$\nabla^2\phi = p_1\phi - p_2'\phi^{3/2} + p_3\phi^2 + (N_{\mathrm{d}} - 1)\,. \tag{32}$$

We use the standard reductive perturbation technique in which the independent variables ξ, η, ζ and τ are stretching as $\xi = \varepsilon^{1/2}(X - V_0 T)$, $\eta = \varepsilon Y$, $\zeta = \varepsilon Z$ and $\tau = \varepsilon^{3/2} T$, V_0 is the phase speed (normalized by c_{sd}) of the wave along the x-direction and ε is a small nonzero constant measuring the weakness of the dispersion. The dependent variables $N_{\mathrm{d}}, V_{\mathrm{d}}, P_{\mathrm{d}}$ and ϕ can be expanded in the power series of ε as

$$N_{\mathrm{d}} = 1 + \varepsilon N_{\mathrm{d}}^{(1)} + \varepsilon^2 N_{\mathrm{d}}^{(2)} + ...\,, \tag{33}$$

$$V_{\mathrm{d}x} = \varepsilon V_{\mathrm{d}x}^{(1)} + \varepsilon^2 V_{\mathrm{d}x}^{(2)} + ...\,, \tag{34}$$

$$P_{\mathrm{d}} = 1 + \varepsilon P_{\mathrm{d}}^{(1)} + \varepsilon^2 P_{\mathrm{d}}^{(2)} + ...\,, \tag{35}$$

$$\phi = \varepsilon\phi^{(1)} + \varepsilon^2\phi^{(2)} + \varepsilon^3\phi^{(3)}....\,, \tag{36}$$

$$V_{\mathrm{d}y,z} = \varepsilon^{3/2} V_{\mathrm{d}y,z}^{(1)} + \varepsilon^{5/2} V_{\mathrm{d}y,z}^{(2)} + ...\,. \tag{37}$$

Using the above stretching coordinate together with the variable expansion of Eqs. (33)-(37) in the normalized basic Eqs. (8)-(10), (32) and collecting the coefficient of the terms of the lowest degree of ε, i.e., $(\varepsilon^{3/2})$, we obtain the same result as Eqs. (18)-(21). However, for the higher degree of ε, i.e., $(\varepsilon^{5/2})$,

$$\frac{\partial N_{\mathrm{d}}^{(1)}}{\partial\tau} - V_0\frac{\partial N_{\mathrm{d}}^{(2)}}{\partial\xi} + \frac{\partial V_{\mathrm{d}}^{(2)}}{\partial\xi} + \frac{\partial\left(V_{\mathrm{d}}^{(1)} N_{\mathrm{d}}^{(1)}\right)}{\partial\xi} + \left(\frac{\partial V_{\mathrm{d}}^{(1)}}{\partial\eta} + \frac{\partial V_{\mathrm{d}}^{(1)}}{\partial\zeta}\right) = 0, \tag{38}$$

$$\frac{\partial V_{\mathrm{d}}^{(1)}}{\partial\tau} - V_0\frac{\partial V_{\mathrm{d}}^{(2)}}{\partial\xi} - V_0 N_{\mathrm{d}}^{(1)}\frac{\partial V_{\mathrm{d}}^{(1)}}{\partial\xi} + V_{\mathrm{d}}^{(1)}\frac{\partial V_{\mathrm{d}}^{(1)}}{\partial\xi} = \frac{\partial\phi^{(2)}}{\partial\xi} + N_{\mathrm{d}}^{(1)}\frac{\partial\phi^{(1)}}{\partial\xi} - \sigma_{\mathrm{d}}\frac{\partial P^{(2)}}{\partial\xi}, \tag{39}$$

$$\frac{\partial P^{(1)}}{\partial\tau} - V_0\frac{\partial P^{(2)}}{\partial\xi} + V_{\mathrm{d}}^{(1)}\frac{\partial P^{(1)}}{\partial\xi} + \frac{5}{3}\frac{\partial V_{\mathrm{d}}^{(2)}}{\partial\xi} + \frac{5}{3}P^{(1)}\frac{\partial V_{\mathrm{d}}^{(1)}}{\partial\xi} + \frac{5}{3}\left(\frac{\partial V_{\mathrm{d}}^{(1)}}{\partial\eta} + \frac{\partial V_{\mathrm{d}}^{(1)}}{\partial\zeta}\right) = 0, \tag{40}$$

$$\frac{\partial^2\phi^{(1)}}{\partial\xi^2} = p_1\phi^{(2)} - p_2'\left(\phi^{(1)}\right)^{\frac{3}{2}} + p_3\left(\phi^{(1)}\right)^2 + N_{\mathrm{d}}^{(2)}, \tag{41}$$

$$\frac{\partial}{\partial\xi}\left(\frac{\partial V_{\mathrm{d}}^{(1)}}{\partial\eta} + \frac{\partial V_{\mathrm{d}}^{(1)}}{\partial\zeta}\right) = \frac{3V_0}{(5\sigma_{\mathrm{d}} - 3V_0^2)}\left(\frac{\partial^2\phi^{(1)}}{\partial\eta^2} + \frac{\partial^2\phi^{(1)}}{\partial\zeta^2}\right). \tag{42}$$

Substituting the values from Eqs. (18) to (21) into Eqs. (38) to (41), followed by a straightforward elimination of $N_{\mathrm{d}}^{(2)}$, $V_{\mathrm{d}}^{(2)}$, $P^{(2)}$, $\phi^{(2)}$, finally we obtain the following new form mK-P equation,

$$\frac{\partial}{\partial\xi}\left(\frac{\partial\phi^{(1)}}{\partial\tau} + A_1'\left(\phi^{(1)}\right)^{\frac{1}{2}}\frac{\partial\phi^{(1)}}{\partial\xi} + A_2\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial\xi} + B\frac{\partial^3\phi^{(1)}}{\partial\xi^3}\right) + C\left(\frac{\partial^2\phi^{(1)}}{\partial\eta^2} + \frac{\partial^2\phi^{(1)}}{\partial\zeta^2}\right) = 0, \tag{43}$$

where the dispersion coefficient B and transverse coefficient C are the same as before and nonlinear coefficients A_1', A_2 are derived as

$$A_1' = \frac{p_2'}{p_1}\frac{\left(3V_0^2 - 5\sigma_{\mathrm{d}}\right)}{4V_0},$$

$$A_2 = \frac{1}{V_0}\left\{\frac{27V_0^2 - 5\sigma_{\mathrm{d}}}{6\left(5\sigma_{\mathrm{d}} - 3V_0^2\right)} + \frac{\left(5\sigma_{\mathrm{d}} - 3V_0^2\right)p_3}{3p_1}\right\},$$

with $p_2' = b_1\mu_{\mathrm{e}}\beta^{3/2}$.

Eq. (42) represents the mK-P equation which describes the nonlinear propagation of DA waves in an electronegative dusty plasma with non-thermal ions and non-isothermal electrons. The required stationary solution of the mK-P equation is

$$\phi^{(1)} = \Big[\frac{4A_1'}{15(Cl^2 + Ul - C)} + \Big\{\frac{16(A_1')^2}{225(Cl^2 + Ul - C)^2}$$

$$+\frac{A_2}{3(Cl^2+Ul-C)}\Big\}^{\frac{1}{2}}\cosh(\frac{\chi}{\omega_2})\Big]^{-2}, \qquad (44)$$

where $\omega_2 = 2\left(\frac{B\,l^3}{C\,l^2+U\,l-C}\right)^{\frac{1}{2}}$.

The above solution of the mK-P equation depends on the nonlinear coefficient A_1' and A_2 where the nonlinear coefficients A_1' and A_2 can be positive, zero or negative depending on the difference of the plasma parameter under different conditions. For negative values of A_2 the above expression gives a complex amplitude, which will not allow the formation of any solitary wave. Therefore the solution of mK-P Eq. (42) gives us compressive solitary waves only when A_2 is positive.

5 Results and discussion

We analytically examine the dependence of the lower degree mK-P, K-P and higher degree mK-P to explain the propagation of an electrostatic solitary wave for various dusty plasma parameters. Here the dust particles in the plasma are considered as uniform in size and negatively charged, while the background plasma includes singly charged positive and negative ions. Fig. 1 shows the effect of the variation of positive ion density ratio $\mu_{\rm p}$ and negative ion density ratio $\mu_{\rm n}$ on the phase velocity V_0. Here, the other plasma parameters are considered as $m_{\rm p}$=40×1.6×10^{-27} kg, $n_{\rm e0}$=4×10^{14} m^{-3}, $n_{\rm p0}$ = 5.4×10^{14} m^{-3}, $n_{\rm n0}=3.0\times10^{14}$, $n_{\rm d0}$=1.2×10^{10} m^{-3}, $Z_{\rm d0}=1.5\times10^4 e$, $T_{\rm e}$=1.5 eV, $T_{\rm i}$ =0.1 eV [9,10]. Fig. 1 clearly depicts that the phase velocity decreases with the enhancement of positive ion density ratio $\mu_{\rm p}$ and negative ion density ratio $\mu_{\rm n}$. In Fig. 2, we depict the effect of the variation of dust and ion temperature ratio $\sigma_{\rm d}$ and ions and electron temperature ratio β on the phase velocity λ. The phase velocity decreases with the dust temperature ratio. Here we consider dust temperature as a constant. That is why the phase velocity decreases with the temperature of the ions. Similarly it is clear from the figure that the phase velocity increases as the ions and electron temperature ratio β decreases. Fig. 3 clearly depicts that the amplitude of lower degree mK-P ϕ_{m1} increases with the enhancement of positive ions density ratio $\mu_{\rm p}$ and negative ion density ratio $\mu_{\rm n}$ It can be concluded from the figure that the lower degree mK-P equation gives the compressive solitary waves whose amplitude increases with the values of positive and negative ions density.

Fig. 4 clearly depicts that the compressive solitary waves profile of the lower degree mK-P equation decreases with the enhancement of positive ions temperature ratio β. Here the positive ions temperature ratio increases since we have kept electron temperature as a constant in the ratio i.e. the ion temperature increases as it reflects more ions. We conclude that by increasing the number of ions, under the same plasmas parameter, the amplitude of the lower degree mK-P equation solution decreases.

Fig. 5 shows the variation of solitary wave profile $\phi^{(1)}$ for the lower degree mK-P equation with the variation of non-thermal ion concentration γ_1 and spatial variable χ. Fig. 5 clearly depicts that, with increasing values of the population of non-thermal ion concentration γ_1, the amplitude of compressive solitary waves from the lower degree mK-P equation decreases.

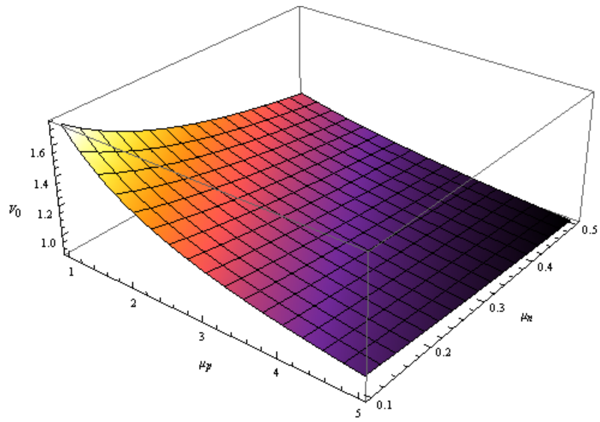

Fig.1 The effect of the variation of positive ion density ratio $\mu_{\rm p}$ and negative ion density ratio $\mu_{\rm n}$ on the phase velocity V_0

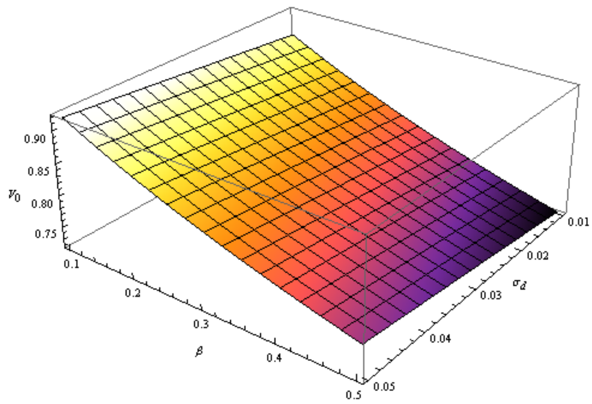

Fig.2 The effect of the variation of dust and ion temperature ratio $\sigma_{\rm d}$ and ions and electron temperature ratio β on the phase velocity V_0

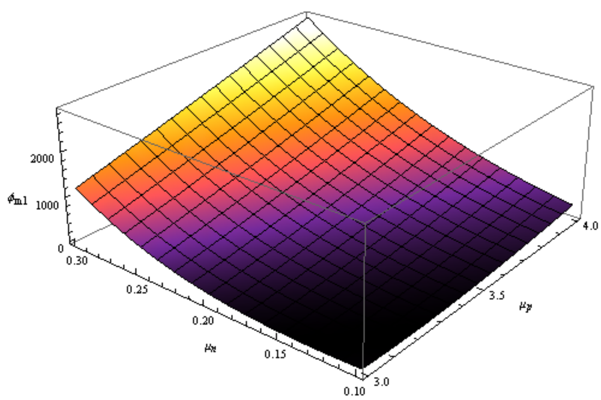

Fig.3 The effect of the variation of positive ion density ratio $\mu_{\rm p}$ and negative ion density ratio $\mu_{\rm n}$ on the amplitude of lower degree mK-P, ϕ_{m1}

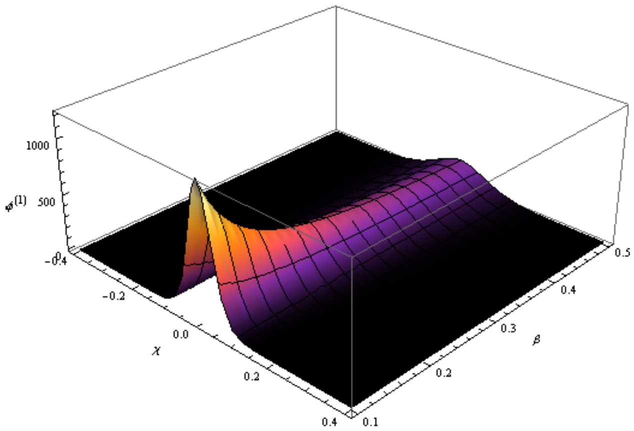

Fig.4 The variation of compressive solitary wave profile $\phi^{(1)}$ with the ion temperature ratio β for the lower degree modified K-P equation

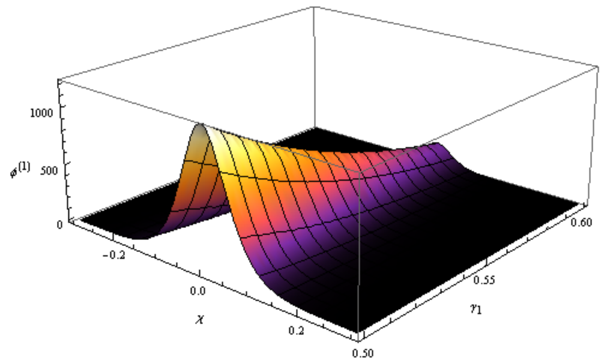

Fig.5 The variation of compressive solitary wave profile $\phi^{(1)}$ with the non-thermal ion concentration γ_1 for the lower degree modified K-P equation

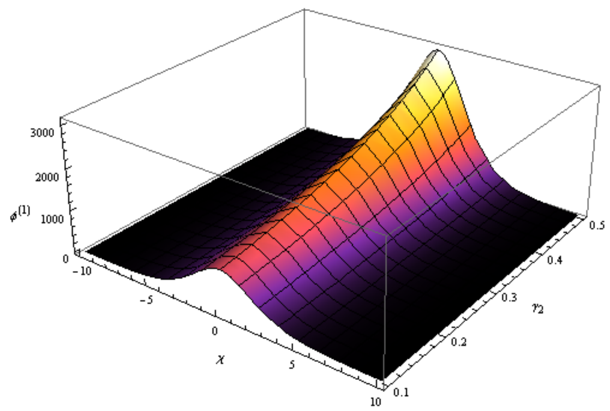

Fig.6 The variation of compressive solitary waves profile $\phi^{(1)}$ with the trapped electron concentration γ_2 for the lower degree modified K-P equation

Fig. 6 shows the variation of solitary wave profile $\phi^{(1)}$ of the lower degree mK-P equation with the variation of non-isothermal electron concentration γ_2. This figure reveals that the amplitude of the compressive solitary wave increases with the enhancement of the non-isothermal electron concentration γ_2. Here the trapped (non-isothermal) electrons temperature increases since we have kept the free electron temperature as a constant in the ratio of the non-isothermal electron concentration γ_2. i.e. by increasing the trapped (non-isothermal) electrons temperature under the same plasmas parameter the solitary waves profile of the lower degree mK-P equation increases.

6 Conclusion

In the present work we have investigated the properties of dust acoustic (DA) solitary waves in a warm adiabatic dusty plasma in the presence of positive and negative non-thermal ions and non-isothermal electrons. The reductive perturbation method is employed to derive the lower degree mK-P equation and the mK-P equation for dust acoustic solitary waves, and the stationary analytical solutions of the lower degree mK-P and mK-P equations are numerically analyzed, and the effect of various dusty plasma constituents on DA solitary wave propagation is taken into account. It is concluded that, both positive, negative ions density ratio, ions temperature ratio, ion concentration γ_1 and electron concentration γ_2 all play an important role in the formation of DA solitary waves. Also we observe that, in the presence of trapped electrons, the compressive solitary wave amplitudes decrease with the enhancement of the number of ions and the compressive solitary wave amplitudes increase with the enhancement of the trapped electrons temperature.

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