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Effect of polarization force on the Jeans instability in collisional dusty plasmas

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Abstract

The Jeans instability in collisional dusty plasmas has been analytically investigated by considering the polarization force effect. Instabilities due to dust-neutral and ion-neutral drags can occur in electrostatic waves of collisional dusty plasmas with self-gravitating particles. In this study, the effect of gravitational force on heavy dust particles is considered in tandem with both the polarization and electrostatic forces. The theoretical framework has been developed and the dispersion relation and instability growth rate have been derived, assuming the plane wave approximation. The derived instability growth rate shows that, in collisional dusty plasmas, the Jeans instability strongly depends on the magnitude of the polarization force.

Keywords: dusty plasmas, Jeans instability, instabilities in plasmas

(Some figures may appear in colour only in the online journal)

1. Introduction

Dusty plasmas, or complex plasmas, comprise heavy dust grains, electrons and ions [1]. The charged heavy grains are often in the 1 pm to 1 cm size range. Dusty plasmas can be found nearly everywhere in space, such as in comet tails and planetary rings [2], as well as in many industrial and laboratory plasmas, for instance laser-driven ablation and tokamak plasmas [3–6]. In this type of plasma, the dust grains are mostly composed of graphite, silicates and amorphous carbon and their radii are of the order of several nanometers to several micrometers. The electrostatic charge of these heavy dust grains is in the range of several orders of the electronic charge e [7, 8]. These heavy and charged dust grains change the temporal and spatial behaviors of the plasma, which leads to new waves, instabilities, etc [1, 9]. Specifically, the presence of these dust particles plays an important role in wave propagation in plasmas and their associated instabilities. It has already been shown that a large variety of electrostatic [10–12], and electromagnetic [13–16] modes are supported by dusty plasmas which become unstable provided that free energy sources are available [17]. Detailed critical reviews about waves and instabilities in non-gravitating dusty plasmas are reported elsewhere [18, 19].

In space plasmas, the dynamics of the larger objects is effectively controlled by gravitation. However, the dynamics of the electrons and ions is mainly controlled by electromagnetic interactions. It is well established that these two forces can become comparable in micron and submicron sized dust grains [20]. Self-gravitation between the heavy dust grains may lead to the collapse of the dusty plasma, resulting in the formation of massive astrophysical objects. This instability effect was first studied by Jeans (later, the term 'Jeans instability' was coined) [21], and is considered to be responsible for the formation of astrophysical objects. In this process, some tiny disturbance in the uniformly distributed mass, caused by self-gravity between the grains, leads to their further localized condensation. For charged dust grains, the electrostatic force between these grains becomes important, and intriguing deviations from the main processes for neutral grains are expected. In a non-uniform dusty plasma with a density gradient, the existence of an external electric field deforms the Debye sheath of the dust grains (polarization), and the plasma–grain interaction force (polarization force) changes their dynamics [22]. Recently, several authors [2, 23–25] have investigated the self-gravity effects of heavy dust grains that modify the existing waves in collisionless dusty plasmas. Prajapati has studied the self-gravity effect by considering the polarization force between the charged dust grains, and has derived the dispersion relation of the Jeans instability [2]. In that paper, the effect of self-gravity between the dust grains is studied in the case where the polarization force exceeds the electrical force. It is concluded that the polarization force greatly affects the collapse of dust grains with radii of about 10 μm. However, dusty plasmas can be highly collisional—for instance in dense molecular clouds—and therefore investigating the instability of a self-gravitating collisional dusty gas against different types of disturbances is crucial [26]. Shukla and Verheest have investigated the instability of self-gravitating collisional dusty plasmas against electrostatic disturbances by treating the electrostatic, gravitational and drag forces simultaneously [27]. Their analytical results exhibit new classes of dissipative instabilities in collisional dusty plasmas due to the effect of self-gravity between the dust grains. Specifically, they have found two types of purely growing modes whose growth rates strongly depend on the dust-neutral and ion-neutral drags. The two studies mentioned can be combined to investigate the concurrent effects of polarization and collisions between the charged dust grains in dusty plasmas on the Jeans instability.

In this study, the polarization force effect on the Jeans instability in collisional dusty plasmas is studied against low-frequency perturbations (perturbations whose phase velocity is smaller than the electrons' velocity in thermal equilibrium). Employing the plane wave approximation, a dispersion relation is derived for collisional dusty plasmas when the polarization force exists, and it is shown that it significantly affects the growth rate of the unstable modes. We first consider the instability of electrostatic waves whose frequency is much smaller than the electronic and ionic collision frequencies, and where the dynamics of the dust fluid will be predominant. Subsequently, we generalize the analysis by focusing on electrostatic disturbances with an intermediate frequency where the ion dynamics will also be predominant in collisional dusty plasmas.

2. Instability analysis

Let us consider multicomponent collisional dusty plasmas comprising charged dust grains, ions and electrons when the polarization force also exists. For a plasma system in equilibrium, the gravitational and the plasma pressure forces are in balance. The instability of such a system is studied for electrostatic waves whose frequency ω is much smaller than the electronic and ionic collision frequencies (ν_e, ν_i) and their wavelengths are shorter than $V_{te,ti}/\sqrt{\omega\nu_{e,i}}$. V_{te} and V_{ti} are the electron and ion thermal velocities (low phase velocities), respectively. The electrons and ions thermalize rapidly in the wave potential ϕ and the corresponding electron and ion density fluctuations are

$$\delta n_e = \frac{n_{e0} e\phi}{k_B T_e} \text{ and } \delta n_i = -\frac{n_{i0} e\phi}{k_B T_i} \tag{1}$$

where $T_j (j = \mathrm{i, e})$ and $n_{j0} (j = \mathrm{i, e})$ are the temperature and unperturbed number density of the ions and electrons, e is the electronic charge (< 0) and k_B is the Boltzmann constant, respectively. It is shown that the polarization force acts on a charged dust grain according to the following equation [22]

$$\boldsymbol{F}_\mathbf{p} = \frac{-Q^2}{2} \frac{\nabla \lambda_D}{\lambda_D^2} \tag{2}$$

where $Q = -z_d e$ is the total charge of the dust grain and λ_D is the linearized Debye radius [2]

$$\lambda_D = \frac{\lambda_{De}\lambda_{Di}}{(\lambda_{De}^2 + \lambda_{Di}^2)^{1/2}} \tag{3}$$

where λ_{Di} and λ_{De} are the linearized Debye radius of ions and electrons, respectively. The continuity and the motion equations in the presence of the polarization force can respectively be written as

$$\frac{\partial}{\partial t}\delta n_d + n_{d0}\nabla\cdot\boldsymbol{U}_\mathbf{d} = 0 \tag{4}$$

$$\frac{\partial}{\partial t}\boldsymbol{U}_\mathbf{d} = \frac{Z_d e}{m_d}\nabla\phi - \nu_d \boldsymbol{U}_\mathbf{d} - \frac{Q^2}{2m_d}\frac{\nabla\lambda_D}{\lambda_D^2} - \nabla\psi \tag{5}$$

where $\boldsymbol{U}_\mathbf{d}$, m_d, Z_d, ν_d and n_d are the velocity, mass, magnitude of the charge, collision frequency with neutral gas background and number density of the dust grains, respectively. ψ also represents the gravitational potential. The Poisson equations for the overall charge and the gravitational mass are also respectively given by

$$\nabla^2\phi = 4\pi e(\delta n_e + Z_d\delta n_d - \delta n_i) \tag{6}$$

$$\nabla^2\psi = 4\pi G m_d \delta n_d \tag{7}$$

where G is the gravitational constant. As has already been proved [2, 28], the polarization force can be written as

$$\boldsymbol{F}_\mathbf{p} = Q\nabla\phi\chi \tag{8}$$

with

$$\chi = \frac{1}{4}\left(\frac{|Q|\ e}{\lambda_D T_i}\right)\left(1 - \frac{T_i}{T_e}\right). \tag{9}$$

Using equations (8) and (9), equation (5) can be recast as

$$\frac{\partial}{\partial t}\boldsymbol{U}_\mathbf{d} = \frac{Z_d e}{m_d}\nabla\phi(1-\chi) - \nu_d\boldsymbol{U}_\mathbf{d} - \nabla\psi. \tag{10}$$

The differential equations can be solved using the plane wave approximation

$$U_d = \phi = \psi \cong \exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r} - \mathrm{i}\omega t) \tag{11}$$

where ω is harmonic disturbance frequency and $\boldsymbol{k} = k_x\mathbf{i} + k_z\boldsymbol{k}$ is the xz plane wave vector. By substitution of equation (11) into equations (4), (6), (7) and (10), one finds, respectively, the following relations

$$\omega\frac{\delta n_d}{n_{d0}} = \boldsymbol{k}\cdot\boldsymbol{U}_\mathbf{d} \tag{12}$$

$$k^2\phi = -4\pi e(\delta n_e + Z_d\delta n_d - \delta n_i) \tag{13}$$

$$k^2\psi = -4\pi G m_d\delta n_d \tag{14}$$

$$\boldsymbol{U}_\mathbf{d} = \frac{\frac{Z_d e\boldsymbol{k}\phi}{m_d}(\chi - 1) + \boldsymbol{k}\psi}{\omega + \mathrm{i}\nu_d}. \tag{15}$$

Combining equations (12)–(15), the dust acoustic wave dispersion relation in the presence of the self-gravitational, collisional and polarization force effects is obtained:

$$(1-\chi)\left(1+\frac{1}{k^2\lambda_{\mathrm{D}}^2}\right)-\frac{\omega_{\mathrm{pd}}^2}{\omega(\omega+\mathrm{i}\nu_{\mathrm{d}})+\omega_{\mathrm{Jd}}^2}=0 \tag{16}$$

where $\omega_{\mathrm{Jd}}^2=4\pi G n_{\mathrm{d0}} m_{\mathrm{d}}$ and $\omega_{\mathrm{pd}}^2=\frac{4\pi Z_{\mathrm{d}}^2 e^2 n_{\mathrm{d0}}}{m_{\mathrm{d}}}$ are the squares of the Jeans and the dust plasma frequencies, respectively. Equation (16) can be rewritten as

$$\omega(\omega+\mathrm{i}\nu_{\mathrm{d}})+\omega_{\mathrm{Jd}}^2-\frac{\omega_{\mathrm{da}}^2}{(1-\chi)}=0 \tag{17}$$

where $\omega_{\mathrm{da}}^2=\frac{k^2\omega_{\mathrm{pd}}^2\lambda_{\mathrm{D}}^2}{1+k^2\lambda_{\mathrm{D}}^2}$ is the dust acoustic frequency.

Now, we generalize the results for collisional dusty plasmas in the intermediate frequency regime. By intermediate frequency we mean frequency of electrostatic disturbances in the range $\nu_{\mathrm{e}}\ll\omega\ll\frac{k^2V_{\mathrm{te}}^2}{\omega}$ [27]. In this regime, the electron number density is still given by equation (1). However, in the limit of $\omega(\omega+\mathrm{i}\nu_{i,d})\gg k^2V_{\mathrm{ti,td}}$ the dynamics of the dust and the ion fluids must be considered in tandem. Since the gravitational, electrostatic and polarization forces balance the inertial accelerations of the dust and the cold ion fluids, the corresponding momentum and continuity equations can be combined and, using plane wave analysis, the density perturbations for the ion and dust fluids can be obtained [27]. Combining equations (12), (14) and (15), δn_{d} can be obtained as follows:

$$\delta n_{\mathrm{d}}=\frac{Z_{\mathrm{d}}\,e\,k^2(\chi-1)\,n_{\mathrm{d0}}\,\phi}{m_{\mathrm{d}}[\omega(\omega+\mathrm{i}\nu_{\mathrm{d}})+\omega_{\mathrm{Jd}}^2]}. \tag{18}$$

Using this equation, the electron number density (equation (1)), and the ion number density in this frequency regime [23]:

$$\delta n_{\mathrm{i}}=-\frac{n_{\mathrm{i0}}k^2e\phi}{k_{\mathrm{B}}m_{\mathrm{i}}\omega^2} \tag{19}$$

in equation (13), after a little algebra the following dispersion relation will be obtained:

$$(1-\chi)\left(1+\frac{1}{k^2\lambda_{\mathrm{De}}^2}\right)-\frac{\omega_{\mathrm{pi}}^2}{\omega(\omega+\mathrm{i}\nu_{\mathrm{i}})}-\frac{\omega_{\mathrm{pd}}^2}{\omega(\omega+\mathrm{i}\nu_{\mathrm{d}})+\omega_{\mathrm{Jd}}^2}=0 \tag{20}$$

where $\lambda_{\mathrm{De}}^2=\frac{k_{\mathrm{B}}T_{\mathrm{e}}}{4\pi e^2 n_{\mathrm{e}}}$ is the square of the electron Debye radius, $\omega_{\mathrm{pi}}=\left(\frac{4\pi n_{\mathrm{i0}}e^2}{m_{\mathrm{i}}}\right)^{1/2}$ is the plasma–ion collision frequency and ν_{i} is the ion-neutral collision frequency. One must note that reducing the amount of dust grain charge will weaken the polarization force according to equation (8). In the limiting case of zero polarization force, equation (18) will be correctly reduced to the dispersion relation obtained by Shukla and Verheest in the absence of the polarization force in dusty plasmas [27].

3. Instability growth rate analysis

The instability growth rate is obtained by knowing that in the limits of $k^2\lambda_{\mathrm{De}}^2\ll 1$ and $\omega\ll\nu_{\mathrm{i}}$, one can rearrange and approximate equation (20) as

$$(1-\chi)+\frac{\mathrm{i}\omega_{\mathrm{ss}}^2}{\omega\nu_{\mathrm{i}}}-\frac{\omega_{\mathrm{da}}^2}{\omega(\omega+\mathrm{i}\nu_{\mathrm{d}})+\omega_{\mathrm{Jd}}^2}=0 \tag{21}$$

where $\omega_{\mathrm{ss}}=k\lambda_{\mathrm{De}}\omega_{\mathrm{pi}}$ is the Shukla–Silin frequency [11] and $\omega_{\mathrm{da}}=k\lambda_{\mathrm{De}}\omega_{\mathrm{pd}}$ is the dust acoustic frequency [27]. In the limiting situation of $\omega\ll\nu_{\mathrm{d}}$, equation (16) can be further approximated as

$$(1-\chi)+\frac{\mathrm{i}\omega_{\mathrm{ss}}^2}{\omega\nu_{\mathrm{i}}}-\frac{\omega_{\mathrm{da}}^2}{\mathrm{i}\omega\nu_{\mathrm{d}}+\omega_{\mathrm{Jd}}^2}=0. \tag{22}$$

This equation can be rearranged into the form

$$(1-\chi)\mathrm{i}\nu_{\mathrm{i}}\nu_{\mathrm{d}}\omega^2+[(1-\chi)\omega_{\mathrm{Jd}}^2\nu_{\mathrm{i}}-\omega_{\mathrm{ss}}^2\nu_{\mathrm{d}}-\omega_{\mathrm{da}}^2\nu_{\mathrm{i}}]\omega+\mathrm{i}\omega_{\mathrm{ss}}^2\omega_{\mathrm{Jd}}^2=0. \tag{23}$$

By solving this quadratic equation and finding its imaginary roots, the instability growth rate of the system can be obtained as follows:

$$\gamma=\frac{1}{2(1-\chi)\nu_{\mathrm{i}}\nu_{\mathrm{d}}}\times\left\{\begin{matrix}(1-\chi)\omega_{\mathrm{Jd}}^2\nu_{\mathrm{i}}-\omega_{\mathrm{ss}}^2\nu_{\mathrm{d}}-\omega_{\mathrm{da}}^2\nu_{\mathrm{i}}+\\ \sqrt{[(1-\chi)\omega_{\mathrm{Jd}}^2\nu_{\mathrm{i}}-\omega_{\mathrm{ss}}^2\nu_{\mathrm{d}}-\omega_{\mathrm{da}}^2\nu_{\mathrm{i}}]^2+4(1-\chi)\nu_{\mathrm{i}}\nu_{\mathrm{d}}\omega_{\mathrm{ss}}^2\omega_{\mathrm{Jd}}^2}\end{matrix}\right\}. \tag{24}$$

Again in the limiting case of $\chi=0$, where the polarization force is absent, the instability growth rate correctly reduces to its counterpart in the Shukla *et al* work, namely [27]

$$\gamma=\frac{1}{2\nu_{\mathrm{i}}\nu_{\mathrm{d}}}\{\omega_{\mathrm{Jd}}^2\nu_{\mathrm{i}}-\omega_{\mathrm{ss}}^2\nu_{\mathrm{d}}-\omega_{\mathrm{da}}^2\nu_{\mathrm{i}}+\sqrt{[\omega_{\mathrm{Jd}}^2\nu_{\mathrm{i}}-\omega_{\mathrm{ss}}^2\nu_{\mathrm{d}}-\omega_{\mathrm{da}}^2\nu_{\mathrm{i}}]^2+4\nu_{\mathrm{i}}\nu_{\mathrm{d}}\omega_{\mathrm{ss}}^2\omega_{\mathrm{Jd}}^2}\}. \tag{25}$$

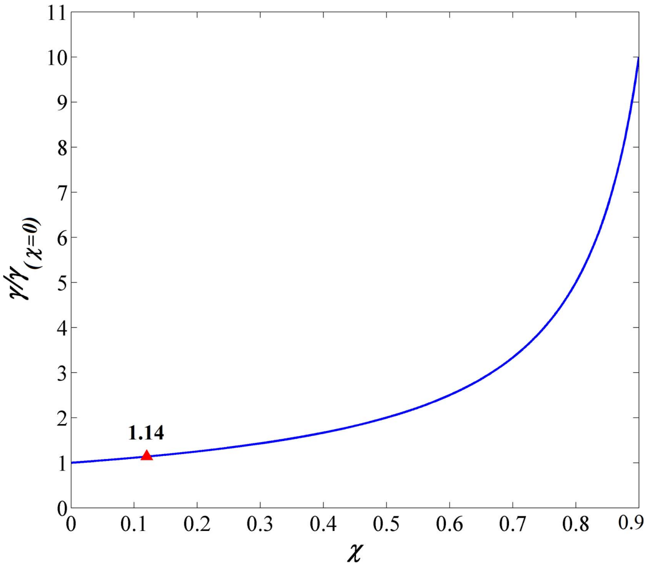

Figure 1. The effect of increasing the polarization force on the growth rate of Jeans instability.

In the other interesting limit of $\omega \gg \nu_d$, the resulting cubic polynomial in ω from equation (16) cannot be solved analytically and one must resort to numerical analysis methods in order to obtain the instability growth rate.

To evaluate the influence of polarization force on the instability growth rate, its normalized values are plotted against χ in figure 1. According to equation (8), χ determines the strength of the polarization force. Normalization is performed by dividing the calculated instability growth rates from equation (24) by the instability growth rate calculated from equation (25) in the absence of polarization force ($\chi = 0$). For numerical calculations, the values of T_i, T_e, ν_i, ν_d, n_e and n_{i0} are also, respectively, chosen as 0.03 eV [2, 29], 3 eV [28], 0.2 s^{-1} [30], 4.6 s^{-1} [31], 1000 cm^{-3} [29] and 2×10^{-3} cm^{-3} [29, 32]. The triangle on the curve shows the normalized instability growth value for the typical values of Q, λ_D and T_i equal to $-10^3 e$, 10^{-2} cm and 0.03 eV, respectively, which leads to $\chi = 0.12$ [2, 28]. Figure 1 indicates that when the polarization force increases, the contraction rate of the plasma, and hence the instability growth rate, also increase.

4. Conclusions

In short, a dispersion relation is analytically derived for collisional dusty plasmas in the presence of a polarization force, and the Jeans instability growth rate is derived. The effects of gravitational force and the collision frequency of the dust grains on the instability growth rate are studied. The results of the present study can be used to gain a proper understanding of the Jeans instability in collisional dusty plasmas when a polarization force exists due to the presence of an external electrical field. In future studies, our analysis can be extended further, to include the magnetic field and electromagnetic effects.

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