Characteristics of dust acoustic waves in dissipative dusty plasma in the presence of trapped electrons

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Abstract

The formation and propagation of nonlinear dust acoustic waves (DAWs) as solitary and solitary/shock waves in an unmagnetized, homogeneous, dissipative and collisionless dusty plasma comprising negatively charged micron sized dust grains in the presence of free and trapped electrons with singly charged non-thermal positive ions is discussed in detail. The evolution characteristics of the solitary and shock waves are studied by deriving a modified Korteweg–de Vries–Burgers (mKdV–Burgers) equation using the reductive perturbation method. The mKdV–Burgers equation is solved considering the presence (absence) of dissipation. In the absence of dissipation the system admits a solitary wave solution, whereas in the presence of dissipation the system admits shock waves (both monotonic and oscillatory) as well as a combination of solitary and shock wave solutions. Standard methods of solving the evolution equation of shock (solitary) waves are used. The results are discussed numerically using standard values of plasma parameters. The findings may be useful for better understanding of formation and propagation of waves in astrophysical plasma.

Keywords: dust acoustic shock wave, trapped electrons, dissipation

(Some figures may appear in colour only in the online journal)

1. Introduction

The characteristics of nonlinear electrostatic waves, especially in dusty plasma in different conditions, have been of great interest and also well explored by the research community worldwide during the last few decades [1–10]. It is well known that, as and when the dissipative term comes into a balanced condition with the nonlinear and dispersive term in nonlinear wave propagation in a plasma medium, one encounters the shock wave phenomenon or otherwise the solitary wave phenomenon [11–14]. The kinematic viscosity, ion-neutral collisions, dust-neutral collisions etc, which are basically wave damping factors, are the cause of dissipation, which results in the formation of shock waves in plasma. The effects of this wave damping parameter on the evolution of nonlinear electrostatic waves have been investigated by many researchers. In a recent report, El-Hanbaly et al discussed the effect of trapped ions on the formation of shock waves in dissipative dusty plasma and they discussed the effect of trapped (free) ion temperature, propagation time etc on shock wave formation as well as on the energy of the solitons and electrostatic field of the waves [15]. In another report El-Hanbaly et al [16] discussed the effect of dissipative properties of plasma on the linear and nonlinear properties of dust acoustic solitary waves in the presence of non-thermal ions for a space plasma environment. It was concluded that dissipative parameters (viscosity) and the temperature ratio of ions to electrons plays an
important role in controlling the behavior of the monotonic oscillatory shock wave phenomenon as well as the solitons. They also discussed the variation of amplitude, width, energy and electrostatic field of the solitons with the non-thermality of the ions and the temperature ratio of ions and electrons. Alam et al [17], considering superthermal dusty plasma, studied the effect of bi-kappa distributed electrons on shock wave formation and concluded that there was a possibility of smoother and weaker compressive and rarefactive shock waves with increasing dissipation. They also observed that with an increase (decrease) of speed of the wave, the amplitude (width) of the shock waves decreases (increases). On the other hand, Mandal et al [18] studied ion acoustic shock waves in a five-component dusty plasma and concluded that the polarity of shock waves was controlled by the ratio of electron-to-positive ion number densities. Shahmansouri [19] studied dissipative dust acoustic solitary waves in plasmas having superthermal ions, and it was concluded that with the superthermal ions approaching a critical value the polarity of the solitary wave changes. Michael et al [20] in a recent report discussed the effect of kappa distributed electrons and ions on shocks/solitons in a dissipative five-component plasma and came to the conclusion that the spectral index of the distribution of electrons and ions controls the transformation of the solitary wave into a shock wave for this type of plasma. Hossen et al [21] discussed the heavy ion acoustic solitary and shock waves in non-thermal adiabatic plasma and found a strong dependence of potential profiles of shock and solitary waves on different dissipative plasma parameters. Pakzad [22] discussed the renovation of solitonic characteristics of plasma to shock-like character in the presence of superthermal electrons and positrons. Borhanian discussed the evolution of ion acoustic solitary and shock waves in dissipative plasma in presence of superthermal electrons deriving cubic complex Ginzburg–Landau type equation [23]. Last but not least, El-Hanbaly [24] recently discussed electron acoustic (EA) soliton, periodic and shock waves in dissipative plasmas having q-non-extensive electron velocity distribution and concluded that the steepness and the amplitude of EA shock waves as well as the sensitiveness of existence of the solitonic regime depends on the non-extensive parameter of hot electrons, hot and cold electron temperatures, density ratio and kinematic viscosity of the plasma medium.

On the other hand, the tremendous application poten-
tialities of non-thermal plasma as a whole in the laboratory, in space, and in different industrial areas has been well studied. Actually, in space and laboratory plasmas, it has been established that negatively charged dust grains and two different temperatures (cold and hot) of non-thermal ions are the major plasma species [25]. In fact, motivated by observations of space plasma, which indicated the presence of non-thermal ion populations [26, 27], an automatic space plasma experiment with a rotating analyzer (ASPERA) on the Phobos satellite detected the loss of energetic ions from the upper ionosphere of Mars. Moreover, we also know electron trapping is inevitable in space as well as in laboratory plasma [28, 29], and researchers throughout the world have been working on the theoretical aspects [30–32] of this trapped state since the early 1970s and experimental aspects [33–35] since the early 1980s. Recently Dev et al [36] discussed the behavior of both compressive and rarefactive shock waves by deriving the 3D Burgers equation with Maxwellian electrons and non-thermal ions. They also pointed out that the evolution of only rarefactive shock waves was possible due to the effect of trapped ions and Maxwellian electrons. Very recently Deka [37] discussed the effect of trapped electrons and non-thermal ions on dust acoustic solitary waves and concluded that only compressive dust acoustic solitary waves exist, due to possible interplay between non-thermality and the trapped state of the plasma. The aim is to open up this exciting area for our companion researcher to verify the theoretical findings, by either observing this type of plasma in space where such plasma species can coexist or creating such a plasma environment in a well controlled laboratory situation, which would be very interesting to see.

2. Theoretical formulation

In this study, we consider an unmagnetized, homogeneous, dissipative and collisionless dusty plasma system for the DAW that comprises variable dust charge, non-thermal ions and a combination of free and trapped electrons. The governing equations for ions and dust are given by

\[
\begin{align*}
\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) &= 0 \\
\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} &= -\frac{\partial \Phi}{\partial x} + \eta \frac{\partial^2 u_i}{\partial x^2} \\
\frac{\partial n_d}{\partial t} + \frac{\partial (n_d u_d)}{\partial x} &= 0 \\
\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} &= -\frac{\partial \Phi}{\partial x} + \eta \frac{\partial^2 u_d}{\partial x^2}
\end{align*}
\]  

(1)

(2)

where \( Q \) is the ratio of dust mass \( m_d \) to the dust charge \( z_d e \).

The system is supplemented by Poisson’s equation as

\[
\frac{\partial^2 \Phi}{\partial x^2} = ne + z_d n_d - n_i.
\]  

(3)

The electron distribution for free and trapped electrons is given by the well known expression [30–32]

\[
n_e = 1 + \phi - \frac{4}{3} \beta \phi^2 + \frac{1}{2} \phi^3 + ...\]

(4)

where \( b = \frac{1 - \beta}{\sqrt{\pi}}, \beta = \frac{T_{df}}{T_{et}}, \) and \( T_{df} \) and \( T_{et} \) are the constant temperatures of free and trapped electrons.

The densities of electrons \( (n_e) \) and ions \( (n_i) \) are normalized to \( n_{e0}, n_{i0} \) respectively. The dust density \( n_d \) and dust grain-charge number are normalized to \( n_{d0}, z_{d0} \) respectively.

The space variable \( x \) is normalized to \( \lambda_{DA} = \left( T_{e} / 4\pi n_{e0} z_{d0} e^2 \right)^{\frac{1}{2}}, \) time \( t \) is normalized to \( \omega_{pe}^{-1} = \left( m_d / 4\pi n_{e0} z_{d0} e^2 \right)^{\frac{1}{2}} \) and the electrostatic wave potential is normalized to \( T_{e} / e \), where \( e \) is the electron charge. The coefficient of viscosity \( \eta \) is a normalized quantity given by \( \omega_{pe}^{-1} \lambda_{DA}^2 m_d n_{e0}. \) All the symbols have their usual meanings.
The charge equation is given\(^2\) by

\[ I_c + I_i = 0 \tag{5} \]

where

\[ I_c = A_1 n_e \exp(\chi_0 z_d), \quad I_i = A_2 n_i (1 - y_0 z_d), \]

\[ A_1 = \frac{\pi a_0^2 n_{do}}{m_e} \left( \frac{8T}{m_e} \right)^{\frac{3}{2}}, \quad A_2 = \frac{\pi a_0^2 n_{do}}{\omega_{pd}} \left( \frac{8T}{m_1} \right)^{\frac{3}{2}}, \]

\[ x_0 = -\frac{e^2 \gamma z_d \sigma_0}{a_d T}, \quad y_0 = -\frac{e^2 \gamma z_d \sigma_0}{a_d T}, \]

\[ \omega_{pd}^{-1} = \left( \frac{m_d}{4\pi^2 a_0^2 n_{do} e^2} \right)^\frac{1}{2}, \quad \sigma_1 = \frac{T_i}{T_e} \]

with \( T_c, T_i \) and \( m_e, m_i \) being the temperatures and masses of electrons and ions; \( a_d \) and \( m_d \) are the radius and mass of the dust particles.

3. Derivation of mKdV–Burgers equation

To study the nonlinear dynamical features of DAW, we employ the standard reductive perturbative method, which is very useful for small amplitude nonlinear waves. This leads to a scaling of independent variables through the stretched coordinates

\[ \xi = \epsilon^\frac{1}{2}(x - vt), \quad \tau = \epsilon^\frac{1}{2} t, \quad \eta = \epsilon^\frac{1}{2} \eta_0 \tag{6} \]

where \( \epsilon \) is a small dimensionless quantity measuring the strength of the dispersion, and \( v \) is the phase velocity along the \( x \) direction in the moving frame of reference given as

\[ v^2 = \left( \frac{n_{do} + \frac{n_{do}}{Q}}{} \right) \]

We can expand the perturbed quantities as follows:

\[ \begin{align*}
 n_1 &= n_{do} + \epsilon \tilde{n}_{d1} + \epsilon^2 n_{i1} + \epsilon^3 n_{i3} + \ldots \\
 n_4 &= n_{do} + \epsilon \tilde{n}_{d4} + \epsilon^2 n_{i4} + \epsilon^3 n_{i3} + \ldots \\
 u_1 &= u_{i1} + \epsilon \tilde{u}_{d1} + \epsilon^2 u_{i3} + \epsilon^3 u_{i3} + \ldots \\
 u_4 &= u_{i4} + \epsilon \tilde{u}_{d4} + \epsilon^2 u_{i4} + \epsilon^3 u_{i4} + \ldots \\
 \phi_1 &= \epsilon \phi_1 + \epsilon^2 \phi_2 + \epsilon^3 \phi_3 + \ldots \\
 z_1 &= 1 + \epsilon z_{d1} + \epsilon^2 z_{d2} + \epsilon^3 z_{d3} + \ldots 
\end{align*} \tag{7} \]

The physical quantities in equations (1)–(3) are expanded by using the expressions given in equations (6) and (7). Equating the lowest order of \( \epsilon \), the continuity and momentum equation give the following results:

\[ \begin{align*}
 n_{i1} &= \frac{n_{do} \phi_1}{v^2}, \quad n_{d1} = -\frac{n_{do} \phi_1}{Q v^2}, \quad u_{i1} = \frac{\phi_1}{v}, \quad u_{d1} = -\frac{\phi_1}{Q v^2}. \\
 z_{d1} &= \frac{\gamma z_{d1} - x_0}{v^2(x_0 - 1 + \gamma y_0)} \phi_1. 
\end{align*} \tag{8} \]

Poisson’s equation gives

\[ \frac{\partial^2 \phi_1}{\partial \xi^2} = \phi_2 - \frac{4}{3} b \phi_1^2 + n_{do} z_{d2} + n_{d2} - n_{i2}. \tag{9} \]

For next higher order of \( \epsilon \), the continuity and momentum equation give

\[ \begin{align*}
 v \frac{\partial n_{i1}}{\partial \tau} - v \frac{\partial n_{i2}}{\partial \xi} + \frac{\partial}{\partial \xi}(n_{do} u_{i2}) &= 0 \tag{10} \\
 v \frac{\partial u_{i1}}{\partial \tau} - v \frac{\partial u_{i2}}{\partial \xi} - \eta_0 \frac{\partial^2 u_{i2}}{\partial \xi^2} &= \frac{4}{3} b \phi_1^2 - \gamma n_{do} z_{d2} \\
 v \frac{\partial n_{d1}}{\partial \tau} - v \frac{\partial n_{d2}}{\partial \xi} + \frac{\partial}{\partial \xi}(n_{do} u_{d2}) &= 0 \tag{12} \\
 v \frac{\partial u_{d1}}{\partial \tau} - v \frac{\partial u_{d2}}{\partial \xi} &= \frac{1}{Q} \frac{\partial \phi_2}{\partial \xi} + \eta_0 \frac{\partial^2 u_{d2}}{\partial \xi^2}. \tag{13} \]

Also, the charge equation (5) gives

\[ x_0 z_{d2} + (1 + x_0) \phi_2 - (1 + x_0) \frac{4}{3} b \phi_1^2 - \gamma n_{do} z_{d2} + (1 - y_0) n_{i2} = 0, \quad \gamma = \left( \frac{T_{me}}{T_{ei}} \right)^\frac{3}{4}. \tag{14} \]

Equation (14) leads to an expression for \( z_{d2} \) as

\[ z_{d2} = \left( \frac{1 - y_0}{v^2} \frac{n_{do}}{x_0 - \gamma n_{do}} \phi_2 \right) + \frac{4}{3} b \phi_1^2 - \frac{4}{3n_{do} (x_0 - \gamma n_{do})} \phi_1^2 - \frac{(1 - y_0) n_{do} n_{do} y_0 \partial \phi_1}{(x_0 - \gamma n_{do}) v^2 \partial \eta}. \tag{15} \]

Now eliminating \( n_{d2}, u_{d2}, n_{i2}, u_{i2} \) from equations (10)–(13) and using the value of \( z_{d2} \), we have the mKdV–Burgers equation as

\[ \frac{\partial \phi_1}{\partial \tau} + \alpha \frac{\partial^3 \phi_1}{\partial \xi^3} + \theta \frac{1}{v^2} \frac{\partial \phi_1}{\partial \xi} + \delta \frac{\partial \phi_2}{\partial \xi} + \kappa \frac{\partial^2 \phi_2}{\partial \xi^2} = 0 \tag{16} \]

where

\[ \alpha = \frac{1}{2b}, \quad \beta = -\frac{bn_{do}(1 + x_0)}{x_0 - \gamma n_{do}}, \]

\[ \delta = -\frac{1}{2} \left( \frac{n_{do} (1 - y_0) n_{do}^2 v^2 - (1 + x_0)}{v^2 (x_0 - \gamma n_{do})} - 1 \right), \]

\[ \kappa = \frac{(1 - y_0) n_{do} y_0}{2 v^2 (x_0 - \gamma n_{do})}. \]

4. Results and discussion

In this section, we represent a detailed discussion on a different type of solution of the mKdV–Burgers equation (equation (16)), and based on the derived solutions we discuss the importance of different physical parameters to the evolution of the solitary and shock phenomenon for the plasma system under consideration.

To solve the mKdV–Burgers equation, i.e. equation (16), we use the substitution \( \chi = \xi - u_0 \tau \), and then the equation
reduces to
\[-u_0 \frac{d\phi_1}{d\chi} + \theta \frac{d\phi_1}{d\chi} + \frac{\kappa}{2\chi} \frac{d^2 \phi_1}{d\chi^2} + \frac{\kappa}{2}\frac{d^2 \phi_1}{d\chi^2} + \frac{\kappa}{\alpha} \frac{d^2 \phi_1}{d\chi^2} = 0. \tag{17}\]

Now neglecting the second order perturbed quantities of equation (17), we obtain
\[-u_0 \frac{d\phi_1}{d\chi} + \theta \frac{d\phi_1}{d\chi} + \frac{\kappa}{\alpha} \frac{d^2 \phi_1}{d\chi^2} = 0. \tag{18}\]

Equation (18) admits two types of solution as discussed below.

4.1. Solitary wave solution

Integrating equation (18) once, we obtain
\[\frac{d^2 \phi_1}{d\chi^2} + \frac{\kappa}{\alpha} \frac{d \phi_1}{d\chi} + \frac{2\theta}{3\alpha} \frac{\phi_1}{\alpha} - u_0 \phi_1 = 0. \tag{19}\]

Now equation (19) can be used to describe a homogeneous and dissipative dusty plasma owing to the presence of the Burgers term (second term of equation (19)). Thus, the trajectory of equation (19) is no longer a simple function of energy, but rather, due to the presence of dissipation, the spatial rate of change of energy will be reasonable to deal with. Thus, equation (19) can be expressed in the general form
\[\frac{d^2 \phi_1}{d\chi^2} + h \left( \phi_1, \frac{d\phi_1}{d\chi} \right) \frac{d\phi_1}{d\chi} + G(\phi_1) = 0 \tag{20}\]
where \( h \left( \phi_1, \frac{d\phi_1}{d\chi} \right) \) is the generalized energy term and \( G(\phi_1) \) is
\[= \frac{u_0}{\alpha} \phi_1. \]

In the conservative case \( h = 0 \), the total energy associated with equation (20) is
\[H = \frac{1}{2} \left( \frac{d\phi_1}{d\chi} \right)^2 + V(\phi_1). \tag{21}\]

Here \( V(\phi_1) \) is the potential function and we have
\[\frac{dH}{d\chi} = \frac{d\phi_1}{d\chi} \left( \frac{d^2 \phi_1}{d\chi^2} + \frac{dV}{d\phi_1} \right). \tag{22}\]

If \( \frac{dV}{d\phi_1} = G(\phi_1) \) in addition to equation (20), the total derivative of \( H \) will be
\[\frac{dH}{d\chi} = -h \left( \phi_1, \frac{d\phi_1}{d\chi} \right) \left( \frac{d\phi_1}{d\chi} \right)^2. \tag{23}\]

If \( h > 0 \), then it is a decreasing function for the variable \( \chi \). Equation (23) is very important as it defines the stability of the system. Here, the term \( \frac{dH}{d\chi} \) corresponds to the Burgers term in the mKdV–Burgers equation (16), as
\[\frac{dH}{d\chi} = \frac{\kappa}{\alpha} \left( \frac{d\phi_1}{d\chi} \right)^2. \tag{24}\]

Thus, as in the present plasma system, the energy is not conserved, so it is difficult to find an exact analytical solution of the mKdV–Burgers equation. In terms of the viscosity coefficient \( \eta_0 \), equation (24) can be written as
\[\frac{dH}{d\chi} = \frac{(1 - \eta_0) \eta_0}{\alpha} \phi_1^2 \tag{25}\]

If the Burgers coefficient \( \kappa = 0 \), equation (19) becomes conservative and consequently \( \frac{dH}{d\chi} = 0 \), then \( H \) reduces to the form
\[H = \frac{1}{2} \left( \frac{d\phi_1}{d\chi} \right)^2 + \frac{4\theta}{15\alpha} \frac{\phi_1^2}{\alpha} - \frac{u_0}{2\alpha} \phi_1^2 \tag{26}\]

where the potential function \( V(\phi_1) \) is given by
\[V(\phi_1) = \frac{4\theta}{15\alpha} \phi_1^2 - \frac{u_0}{2\alpha} \phi_1^2. \tag{27}\]

Since \( \frac{d^2 V}{d\phi_1^2} = -\frac{u_0}{\alpha} < 0 \) for positive values of \( u_0 \) and \( \alpha \), the stable solitonic solution is given by
\[\phi_1 = \phi_0 \sech^2 \left( \frac{\chi}{W} \right) \tag{28}\]

Figure 1. Variation of potential profile of solitary wave with different free to trapped electron temperature ratios \( \beta \).
species towards Boltzmannian thermal equilibrium. It is seen
that the amplitude of the compressive solitons increases with
negatively decreasing value of $\beta$. Here we have considered a
negative value of $\beta$ following a depression in the distribution
of the trapped particles. Physically, more and more depression
in the particles’ distribution will eventually result in a
reduction in the amplitude of the wave as the plasma system
advances in the direction of equilibrium as $\beta$ approaches 0
and further towards a positive value (to be shown in the later
part of this paper). Here and throughout the paper, the basic
plasma parameters considered in the numerical analysis are
adopted from [4–6] as well as [8–10] along with [37].

In figure 2, the spatial propagation of the soliton pulse at
different time scales $\tau$ is presented. It is seen that, as the
propagation time increases, the solitary wave also becomes
faster and as a result its amplitude increases considerably.
This may be because, in the absence of any kind of dissipa-
tion, the wave gains amplitude with elapsing time. In figure 3,
the evaluation of the amplitude $\phi_0$ is shown with $\beta$ and $\sigma_i$.
It can be observed that the amplitude of the soliton increases
with increasing value of both $\beta$ and $\sigma_i$. The fluctuation of the
width $W$ with $u_0$ is plotted in figure 4. It emphasizes that the
soliton width increases with decreasing value of $u_0$; this is
obvious because, with increasing (decreasing) speed of the
wave, the wave will show an enhancement (diminution) in the
amplitude, which results in a decrease (increase) in the width
of the wave.

Moreover, the soliton energy $E_n$ of the wave is given by

$$E_n = \int_{-\infty}^{\infty} u_d(x) \, dx.$$  \hspace{1cm} (29)

Integrating equation (29) gives

$$E_n = \frac{900 u_0^2}{448 \theta^2 Q \gamma^2} \sqrt{\frac{\alpha}{\gamma}},$$

which can be calculated based on mainly plasma parameters. The
changes of soliton energy $E_n$ with $\sigma_i$ and $\beta$ is plotted in
figure 5.

It is seen that the soliton energy increases with increasing
values of both $\sigma_i$ and $\beta$. This may be because in the present
system, with increasing $\sigma_i$ i.e. with increasing thermal velo-
city, the soliton will gain energy and also with increasing $\beta$,
which physically admits the movement of the system towards
becomes dissipative and consequently the total energy $H$ of the system is not conservative. In such a case, it is difficult to find out the exact solution of equation (19). Nowadays many researchers use the powerful tangent hyperbolic method (tanh) to solve nonlinear differential equations of this type and for this the transformation $\mu = \tanh(\chi)$ is introduced. Then equation (19) reduces to

$$
(1 - \mu^2)\frac{d^2\phi_1}{d\mu^2} - \left(2\mu - \frac{\kappa}{\alpha}\right)\frac{d\phi_1}{d\mu} + \frac{2\theta}{3\alpha} \mu^2 - \frac{u_0}{\alpha} \phi_1 = 0.
$$

(32)

To find the series solution of equation (32), we follow the technique of the tanh method and consider the solution of the form

$$
\phi_1(\mu) = \sum_{r=0}^{\infty} a_r \mu^{r+r}.
$$

(33)

To find the finite solution of equation (33), the values of $r$ and $\rho$ should be determined. Substituting equation (33) in equation (32) and to leading order, analysis of finite terms gives $r = 4$ and $\rho = 0$. Then $\phi_1(\mu)$ becomes

$$
\phi_1(\mu) = a_0 + a_1 \mu + a_2 \mu^2 + a_3 \mu^3 + a_4 \mu^4.
$$

(34)

Using the expression for $\phi_1(\mu)$ in equation (32) and balancing the nonlinear and dispersion terms, we obtain $a_3 = 0$ and $a_4 = 0$. Moreover, the values of other coefficients can be obtained from the following algebraic equations.

$$
\frac{u_0}{\alpha} a_0 + \frac{2\theta}{3\alpha} a_0^\frac{3}{2} - \frac{\kappa}{\alpha} a_1 + 2a_2 = 0,
$$

(35a)

$$
-2a_1 - \frac{u_0}{\alpha} a_1 - \frac{\theta}{\alpha} \sqrt{a_0} a_1 + \frac{2\kappa}{\alpha} a_2 = 0,
$$

(35b)

$$
\frac{\kappa}{\alpha} a_1 + \frac{\theta}{4\alpha} a_1^2 - 8a_2 - \frac{u_0}{\alpha} a_2 - \frac{\theta}{\alpha} \sqrt{a_0} a_1 = 0,
$$

(35c)

$$
2a_1 - \frac{2\kappa}{\alpha} a_2 + \frac{\theta}{2\alpha} a_1 a_2 = 0,
$$

(35d)

$$
6a_2 + \frac{\theta}{4\alpha} a_2^2 = 0.
$$

(35e)

Solving this set of algebraic equations, one obtains

$$
a_0 = \frac{(u_0 + 12\alpha)^2}{\theta^2},
$$

$$
a_1 = \frac{24\kappa(u_0 + 12\alpha)}{5\theta^2}
$$

and

$$
a_2 = -\frac{24\alpha(u_0 + 12\alpha)}{\theta^2}.
$$

(36)

With the help of the above expressions, the solution of the mKdV–Burgers equation can be modified as

$$
\phi_1 = \frac{(u_0 + 12\alpha)^2}{\theta^2} \left[ u_0 - 12\alpha + \frac{24\kappa}{\alpha} \tanh(\chi) + 24\alpha \operatorname{sech}^2(\chi) \right].
$$

(37)
with free to trapped electron temperature ratio of the coefficient of the tanh term compared with the sech terms in it. Structures due to the presence of dispersion and dissipative is found that the solution of equation 4.1.2. Monotonic and oscillatory shock solution. One can find another type of solution of equation (19) if it is considered that the dissipative term (κ) prevails over the dispersive term (α). Then equation (19) is reduced to the following nonlinear form of differential equation:

$$\frac{d\phi_1}{d\chi} + \frac{2(\kappa)}{3\kappa} \frac{\phi_1}{\kappa} - \frac{u_0}{\kappa} \phi_1 = 0. \quad (38)$$

The solution of equation (38) is given by

$$\phi_1 = 9u_0^2 \frac{u_0}{\kappa} \frac{\alpha}{\pi^2} \left( e^{\frac{u_0}{\kappa} \phi_1} + 2\theta e^{\frac{u_0}{\kappa} \phi_1} \right)^2 \quad (39)$$

where c_1 is a constant of integration. Using the boundary condition \(\chi \to -\infty\), \(\frac{d\phi_1}{d\chi} \to 0\), one obtains \(c_1 = 0\) and then the solution equation (39) reduces to

$$\phi_1 = 9u_0^2 \frac{u_0}{\kappa} \frac{\alpha}{\pi^2} \left( 1 + 2\theta e^{\frac{u_0}{\kappa} \phi_1} \right)^2 \quad (40)$$

This shows that the solution equation (40) really depicts monotonic shock waves. Again, if the dispersive term is considered with dissipative term then another type of asymptotic solution can be found using the boundary conditions \(\chi \to \pm\infty\) ⇒ \(\frac{d\phi_1}{d\chi} = \frac{d^2\phi_1}{d\chi^2} = 0\) and \(\chi \to \pm\infty\) ⇒ \(\phi_1 \to \phi_c\), \(\frac{d\phi_1}{d\chi} = \frac{d^2\phi_1}{d\chi^2} = 0\). Now from equation (38) we have \(\frac{2\theta}{3\kappa} \phi_1^2 - \frac{u_0}{\kappa} \phi_1 = 0\), which gives \(\phi_1 = 3u_0/2\theta \phi_c^2\). By using \(\phi_1 = \phi_c + \phi'\), for \(|\phi_c| \gg |\phi'|\), equation (19) can be linearized as

$$\frac{d^2\phi'}{d\chi^2} + \frac{\kappa}{\alpha} \frac{d\phi'}{d\chi} - \frac{u_0}{\alpha} \phi' = 0. \quad (41)$$

The solution of the equation (41) are proportional to \(\exp(p_\alpha)\), where \(p_\alpha\) is given by

$$p_\alpha = \frac{\kappa}{2\alpha} \left[ -1 \pm \sqrt{1 - \frac{4u_0/\alpha}{\kappa^2}} \right]. \quad (42)$$
For $\kappa^2 \ll 4\sigma$, the oscillatory shock wave solution is given by
\begin{equation}
\phi_l = \phi_k + K \exp \left( -\frac{\kappa}{2\chi} \right) \cos \left( \sqrt{\frac{4\sigma}{\chi}} \right) \tag{43}
\end{equation}
where the constant $K = -\phi_k$ (generally). On the other hand, if we do not neglect the second order perturbed quantity of equation (17), then the term $\delta \frac{d\phi_l}{d\chi}$ comes into existence. If it is assumed that $\delta \frac{d\phi_l}{d\chi} \rightarrow \rho \frac{d\phi_l}{d\chi}$ with suitable conditions, where $\rho$ is constant, then equation (17) will reduce to the same type as equation (18) and then it can be solved as above.

5. Conclusion

The formation of nonlinear waves of small amplitude and their propagation in the presence of micro-sized and negatively charged dust particles in homogeneous, collisionless, unmagnetized and dissipative dusty plasma with trapped electrons and containing non-thermal ions are studied in the form of the mKdV–Burgers equation. The mKdV–Burgers equation is derived with the help of the perturbative technique using basic sets of fluid equations.

In the analysis, it is observed that the mKdV–Burgers equation is not easily integrable as the energy system of the plasma is not conservative for the presence of the dissipative Burgers term. If the Burgers term is removed, then the effect of dissipation becomes negligible in comparison to dispersion and nonlinearity. Also, the equation becomes easily integrable and an analytic solution is found in such a case. The behaviors of the solitonic solution, equation (28), in the absence of the Burgers term are discussed using standard values of plasma parameters. In the presence of the Burgers term ($\kappa \neq 0$), strong dissipation arises, which has an effect on the amplitude of the waves. In a condition where the nonlinear term, the dispersive term and dissipative term are balanced, the solution combines the solitary and shock waves. The combination between the soliton and shock waves is obtained by using the well known tanh method, in the solution of equation (37). In a situation where the dispersion is negligible in comparison to dissipation, the nonlinear term and dissipative term are in balance. In this circumstance, the solution of equation (43) gives oscillatory shock waves (not shown here). Finally, it is observed that electron and ion temperature and dust grain charge play an important role in finding the existence of solitary waves; the nonlinear term, dissipation and dispersion also have ineluctable roles in formation of shock waves (both monotonic and oscillatory).

References

[34] Saeki K et al 1979 Phys. Rev. Lett. 42 501