Spatial charge and compensation method in a whirler

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Abstract
Based on particle-in-cell simulation, we studied the motions of ions and electrons. The results have shown that electrons are bounded by a magnetic field and only a small number of electrons can pass through the whirler channel. The plasma becomes non-neutral when it is emitted from the whirler, and the spatial charge leads to a beam divergence, which is unfavorable for mass separation. In order to compensate the spatial charge, a cathode is designed to transmit electrons and the quasi-neutral plasma beam. Experiment results have demonstrated that the auxiliary cathode can obviously improve the compensation degree of the spatial charge.

Keywords: plasma mass separator, whirler, spatial charge, compensation method, PIC

( Some figures may appear in colour only in the online journal)
the boundary conditions as shown in figure 2. In the whirler, the channel insulation wall is made of BN ceramic material, and the collisions between the particles and ceramic material refer to the model of secondary electron emission in [13]; ions and electrons inject from the left side (drive boundary) and disappear from the right side (open boundary). The radial and axial length of the whirler channel are $H (= 1$ cm) and $L (= 3$ cm), respectively. In this simulation, the radial magnetic field $B_r = 0.286$ T, the working medium is argon ($A_r$), and the plasma density is set to $6 \times 10^{11}$ cm$^{-3}$.

The simulated result is shown in figure 3. When a plasma beam enters the whirler channel, the electrons are bounded at the entrance of the whirler by a magnetic field. And the depth of the charged particles (ions or electrons) penetrating the magnetic field depends on their Larmor radius [14]

$$r_L = \frac{v_L}{|\omega_L|} = \frac{mv_L}{|q|B_r}$$  \hspace{1cm} (1)

where $m$, $q$, and $\omega_L$ are the mass, charge, and cyclotron frequency of the charged particles, respectively, and $v_L$ is the velocity perpendicular to the radial magnetic field $B_r$. When the radial magnetic field is 0.286 T, the Larmor radius of the ions is larger than the axial length of whirler channel $L$, and so the ions can pass through the whirler and gain an azimuthal velocity of $v_{iA} = qB_rL/m$ according to the angular momentum conservation. However, the Larmor radius of the electrons is much smaller than $L$, and so the electrons are bounded at the entrance of the whirler and revolve around the magnetic lines. A separation of positive and negative charges is generated, because the motion characteristics of the electrons and ions passing through the magnetic field are different. The axial electric field $E_z$ (shown as figure 4), which is caused by the separation of positive and negative charges, pulls the electrons towards the exit. When $B_r = 0.286$ T, the condition $\omega_{c\epsilon} \tau_e \gg 1$ is satisfied, and the velocity of the electrons perpendicular to the magnetic field of electrons is [14]

$$v_{\epsilon} = \frac{e\nu}{(\nu^2 + \omega_{c\epsilon}^2)m_{\epsilon}}E_z = -\frac{k_B T_e \nu}{(\nu^2 + \omega_{c\epsilon}^2)m_{\epsilon}} \frac{\nabla n_{\epsilon}}{n_{\epsilon}}$$ \hspace{1cm} (2)

where $e$ and $m_{\epsilon}$ are the charge and mass of the electrons, respectively, the electron collision period $\tau_e = \nu^{-1}$, $\omega_{c\epsilon}$ is the electron cyclotron angular frequency (rad $\cdot$ s$^{-1}$), $k_B$ is Boltzmann’s constant, $T_e$ is the electron temperature, $\nu$ is the electron mobility, and $D_\perp$ are the electron mobility and diffusion coefficient perpendicular to the magnetic field, respectively. As indicated by formula (2), the motion of the electrons is bounded by the magnetic field, but the collision contributes to the process of the electrons traversing the magnetic field due to its interruption of the electron Larmor cyclotron. Before a new, stable Larmor cyclotron is established, the electrons have moved a
distance towards the whirler exit under the axial electric field. The classical conduction is caused by collisions between the electrons and heavy particles (atoms and ions). Besides, there is another type of conduction—Bohm conduction caused by plasma turbulence. The electron diffusion coefficient of Bohm conduction is

\[ D_{\text{Bohm}} = \frac{1}{16} \frac{T_e}{eB_z^2} \]  \hspace{1cm} (3)

The velocity of the electrons traversing the magnetic field can be rewritten as

\[ v_e = \mu_{e\perp} E_z - (D_{e\perp} + D_{\text{Bohm}}) \frac{\nabla n_e}{n_e} \]  \hspace{1cm} (4)

where \( E_z \) is the axial electric field strength. Under the effect of axial electric field and \( \nabla n_e \), the electrons transfer towards the whirler exit due to classical conduction and Bohm conduction. Therefore, the electron density \( n_e \) declines with the increase of the axial distance from the entrance of the whirler, and only a small number of electrons can spurt out of the channel. The result of the above theoretical analysis is essentially in agreement with the simulation result shown in figure 3.

The ion density is always larger than the electron density \( n_i > n_e \) at the exit of the whirler, and the plasma beam is non-neutral. Spatial charges will lead to a beam divergence, which is unfavorable for beam focusing and mass separation. In order to compensate the spatial charge, a lanthanum hexaborid (LaB\(_6\)) cathode is placed at the exit to improve the performance of the whirler.

### 3. Experimental research on the spatial charge compensation

The structure of the improved whirler with a LaB\(_6\) cathode placed at the exit is illustrated in figure 5, and the scheme of the LaB\(_6\) cathode is presented in figure 6. As shown in figure 6, the emitter (12) is the core element of the cathode, and LaB\(_6\) is chosen due to its advantages of low emission power, strong electron emission capability, nontoxicity, and simple operation. Tungsten wires (10) are heated by a high current to keep the emitter at 1200 °C, since the emitter is capable of effectively emitting free electrons at this temperature. In the experiment, the working parameters of the LaB\(_6\) cathode are set with the discharge voltage of 400 V, discharge current of 2.17 A, mass flow of 0.3 mg s\(^{-1}\), heating voltage of 12 V, as well as the heating current of 14 A. The external electromagnetic field is set as zero in the focusing device, \( B_z = 0 \) and \( E_z = 0 \). The plasma source accelerating voltage is \( U_{\text{AC}} = 900 \text{ V} \) and the whirler radial magnetic field is \( B_r = 0.286 \text{ T} \). The effect of compensation on the spatial charge is estimated by measuring the ion energy distribution, the radial distribution of argon ions, as well as the position of the argon ions with the most probable energy. And the movable energy analyzer with the retarding potential [15] is used to measure the energy distribution function of ions. This method can exclude the overlapping of the spectra of the plasma beam ions and secondary plasma ions.

The ion divergence in the focusing device is caused by the effect of the centrifugal force and the electric field generated by the spatial charge, and so the ion energy radial distribution is dependent on the radius \( r \) and the non-neutrality parameter \( \varepsilon \), and a large value of \( \varepsilon \) corresponds to low-degree compensation. As a prediction, the relational formula of ion energy \( W \) and radius \( r \) for different compensation degree is derived numerically first. Here, the whirler exit is set as the origin of the cylindrical coordinate system, \( r, \varphi, \) and \( z \) are the radial, azimuthal, and axial coordinate axes, respectively. With the electron zero temperature approximation (real \( T_e \approx 15 \text{ eV} \)), the ion beam under the steady-state conditions can be described by

\[ n_i(v_r, z)2\pi\sigma(r(v_r, z), z)Dr(v_r, v_z + \Delta v_z)v_z = \text{const} \]

\[ = 2\pi HRv_zf_0(v_z)\Delta v_z \]  \hspace{1cm} (5)

where \( n_i(v_r, z) \) is the ion density at the point \((z, r(v_r, z))\); \( \Delta r(v_r, v_z + \Delta v_z) \) is the radial distance between the ions with the velocities \( v_z \) and \( (v_z + \Delta v_z) \); \( f_0(v_z) \) and \( \int_0^{\infty} f_0(v_z)dv_z = n_0 \) are the distribution function and density of ions at the whirler exit \((z = 0)\), respectively.

From formula (5), considering \( \Delta r(v_r, v_z + \Delta v_z) = \frac{dr(v_r, z)}{dv_z}\Delta v_z \), the ion density at the point \((z, r(v_r, z))\) is

\[ n_i[z, r(v_r, z)] = R\overline{H}f_0(z)\left(\frac{\pi\overline{r}(z, v_r)}{\overline{v_r}(z, v_r)}\right)^2. \]

The degree of non-neutrality can be described as \( \Delta n[d(z, r(v_r, z))] = n_1 - n_e = \varepsilon n_1[z, r(v_r, z)] \). For a cylinder with an axial length of \( \Delta z \) and a radius of \( r(v_r, z) \), \( E_{\text{pp}}[z, r(v_r, z)] \)

\[ 2\pi r(z, v_r)\Delta z = 4\pi\varepsilon\Delta z2\pi \int_{v_z^\text{min}}^{v_z^\text{max}} f_0(x)dx \]

where \( E_{\text{pp}} \) is the electric field generated by the spatial charge. In accordance with the Gauss theorem, \( E_{\text{pp}} \) follows that

\[ E_{\text{pp}}[z, r(v_r, z)] = \frac{4\pi\varepsilon\overline{H}r[\Delta e]}{\overline{r}(z, v_r)} \]

\[ = 4\pi\varepsilon\overline{H}r[\Delta e]/\overline{r}(z, v_r) \]

\[ = 4\pi\varepsilon\overline{H}r[\Delta e]/\overline{r}(z, v_r) \]  \hspace{1cm} (6)
where $W = \frac{M v_z^2}{2}$ and $f_0(v_z) = f_0(W)$. The radial motion of ions (with a mass of $M$ and a velocity of $v_z$) can be described by the following equation

$$\frac{d^2r(t, v_z)}{dt^2} = \frac{v_z^2}{r} + \frac{eE_p(r)}{M}. \quad (7)$$

According to the angular momentum conservation, $M v_z r = M v_{zA} R$. Then, the azimuthal velocity of an ion at an arbitrary point is $v_z = v_{zA} R/r$, where $v_{zA}$ is the azimuthal velocity of an ion at the whirler exit. Considering $t = z/v_z$, the definition of $v_{zA}$ and $E_p[z, r(v_z, z)]$ from (6), equation (7) may be written as

$$\frac{d^2\rho}{dx^2} = \frac{1}{\rho^3} + \frac{\gamma\eta(\omega)}{\rho} \eta(\omega) \left\{ \ln \rho^2 \left( \frac{1}{2\sqrt{\rho^2 - 1}} - 1 \right) - \sqrt{\rho^2 - 1} - 2 \arctan \sqrt{\rho^2 - 1} \right\} = x = \frac{\xi}{\sqrt{A\omega}}. \quad (9)$$

For a fixed $\xi$, equation (9) is the relational formula of $\rho$ (corresponding to $r$) and $\omega$ (corresponding to $W$) for different degree compensation. The values of $\gamma$ and $B_{1L}$ can be obtained from experimental data. $\eta(\omega) \rightarrow 0$ when $\omega \rightarrow \omega_{\text{max}} = W_{\text{max}}/eU_{AC}$; therefore, when $\omega$ is close to $\omega_{\text{max}}$, equation (9) may be written as

$$\rho_0(\omega) = \sqrt{1 + \frac{\xi^2}{A\omega}}. \quad (10)$$

Equation (10) works for arbitrary $\omega$ with a completely compensating ion beam corresponding to $\gamma = 0$. As to the compensation experiment, the measuring position is set at a fixed distance of $z = 310 \text{ mm}$ from the whirler exit in the focusing device. Figure 7 shows the energy distribution of krypton, argon, and nitrogen (the curves are labeled ‘Kr’, ‘Ar’, and ‘N’, respectively) ions corresponding to the most probable energy (maximum $|dI/dW|$). Figure 7 shows that ions with maximum energy remain near the axis of the exit of the whirler, and the ion energy $W$ decreases with the increase.

**Figure 6.** The structure of the LaB$_6$ cathode.

**Table 1.** Components of the focusing device.

<table>
<thead>
<tr>
<th>Number</th>
<th>Component</th>
<th>Material</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>Tungsten wire</td>
<td>W</td>
</tr>
<tr>
<td>2</td>
<td>Column</td>
<td>Mo</td>
</tr>
<tr>
<td>3</td>
<td>End cap</td>
<td>Stainless steel</td>
</tr>
<tr>
<td>4</td>
<td>Gland 1</td>
<td>BNC</td>
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<td>Sleeve</td>
<td>Ceramics</td>
</tr>
<tr>
<td>6</td>
<td>Gland 2</td>
<td>BNC</td>
</tr>
<tr>
<td>7</td>
<td>Upper support</td>
<td>BNC</td>
</tr>
<tr>
<td>8</td>
<td>Tantalum skin</td>
<td>Ta</td>
</tr>
<tr>
<td>9</td>
<td>Housing</td>
<td>Stainless steel</td>
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<tr>
<td>10</td>
<td>Tungsten wire</td>
<td>W</td>
</tr>
<tr>
<td>11</td>
<td>Teabowl</td>
<td>Mo</td>
</tr>
<tr>
<td>12</td>
<td>Emitter</td>
<td>LaB$_6$</td>
</tr>
<tr>
<td>13</td>
<td>End socket</td>
<td>Mo</td>
</tr>
</tbody>
</table>
of radius $r$ (at a fixed distance $z$) in the equipotential focusing device. The experimental data agree with those of equation (10). On the other hand, the divergence degree of the plasma beam is larger than the expected value, which means ions of different mass still overlap with each other, leading to an inevitable decrease of separation purity.

Figure 8 presents the radial distribution of argon ions. Compared to curves 4–6, a prominent influence of the spatial charge on the beam divergence can be found in curves 1–3. The ions of the uncompensated beam are obviously diverged by the electric field of the spatial charge, and the diverged degree is more significant than that of the compensated beam of 4–6, which means the uncompensated plasma beam is more difficult to separate.

In figure 9, curve 1 is calculated through equation (10), while curves 2 and 3 are calculated through equation (9). Diamonds and dots represent the experimental data. The compensation degree of the spatial charge can be obtained by comparing the curves and experimental data.

As can be seen in figure 9, curve 3 agrees well with the dots. That means, with regard to the case without compensation, there are only a small number of electrons passing through the whirler, and appearing in the focusing device. The experimental results agree with those obtained from PIC simulation, as shown in figure 3. However, the experimental results with compensation (diamonds in figure 9) are different from the calculations of the complete compensation case ($\gamma = 0$—curve 1 in figure 9), but agree with the calculations of the incomplete compensation case ($\gamma = 0.114$—curve 2 in figure 9). And that is the main reason for larger divergence degree of the plasma beam beyond expectation, as shown in figure 7. In general, the experimental results show that the auxiliary cathode can improve the compensation degree of the spatial charge obviously, but not completely. The exact reason is still uncertain, but is probably due to the complex interactions between the plasma oscillation, magnetic field, and cathode.

It is clear that the complete compensation plasma beam can decrease the beam divergence and improve the mass separation effect. Therefore, in order to achieve a complete compensation for the beam, further research will concentrate on improving the working performance of the auxiliary cathode and increasing the mobility of electrons by optimizing the whirler topography.

4. Conclusion

Based on the PIC simulation and experiments, the spatial charge problem of the plasma beam passing though the whirler and the compensation method are studied. Due to the different Larmor radii, the passing rate of ions is greater than that of electrons, and non-neutrality beams diverge in the focusing device. In order to optimize the performance of the
whirler, a LaB$_6$ cathode is placed at the exit of the whirler to neutralize the beam. The measurement results show that the application of the LaB$_6$ cathode can improve the compensation degree of the spatial charge and enhance the beam focusing and mass separation obviously.

**Acknowledgments**

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**References**