Some studies on transient behaviours of sheath formation in dusty plasma with the effect of adiabatically heated electrons and ions

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Abstract

Based on quasipotential analysis, a plasma sheath is studied through the derivation of the Sagdeev potential equation in dusty plasma coexisting with adiabatically heated electrons and ions. Salient features as to the existence of sheaths are shown by solving the Sagdeev potential equation through the Runge–Kutta method, with appropriate consideration of adiabatically heated electrons and ions in the dynamical system. It has been shown that adiabatic heating of plasma sets a limit to the critical dust speed depending on the densities and Mach number, and it is believed that its role is very important to the sheath. One present problem is the contraction of the sheath region whereby dust grains levitated into the sheath lead to a crystallization similar to the formation of nebulons and are compressed to a larger chunk of the dust cloud by shrinking of the sheath. Our overall observations advance knowledge of sheath formation and are expected to be of interest in astrophasmas.

Keywords: adiabatic plasmas, dusty plasmas, Bohm criterion, sheath

(Some figures may appear in colour only in the online journal)

1. Introduction

Studies of plasma sheaths in space as well as in the laboratory have been of increasing interest to plasma researchers [1–4]. Because of their extensive applications, many models have been developed to exhibit sheaths in industrial plasmas. Dusty plasmas have lately been recognized as having important to the formation of sheaths in laboratory and space plasmas and thus, in continuation, electrostatic sheaths have been studied in plasmas contaminated by dust grains with constant [5] and varying charges [6]. Dust grains, which are ubiquitous by nature throughout plasmas, are charged due to the interaction of electrons, ions, background radiation [7] and interaction exhibits many new features on nonlinear plasma-acoustic modes. Dust grains facilitate many unique phenomena as to the formation of dust crystals, stable dust atmospheres as nebulons, and dust-acoustic modes, and studies have been generated with the concept of fully ionized plasma or plasma contaminated with dust grains found in the laboratory [8–11] as well as in space plasma environments [11–14]. Many salient features of nonlinear plasma acoustic waves such as solitons, double layers, shock waves, and sheath are found in the astrophysical problems of cometary tails, asteroid zones, planetary rings, interstellar medium, and on-Earth environments. Due to the high density of dust grains in these regions, different wave modes of dust acoustic waves [15], dust-ion-acoustic waves [16, 17], and dust lattice waves [18, 19] have been yielded.

Reductive perturbation technique [20] and pseudopotential analysis [21] have provided a unique platform to study nonlinear plasma acoustic waves as well as sheath formation, and have been bridging the gap between the potentiality of the theoretical observations and their realization in laboratory [22] and space plasmas [23–25]. Many researchers [2, 26–28] have extended their works on electrostatic potential in different dusty plasmas with Boltzmann response of ions and electrons. Others have studied the characteristic behaviours of nonlinear phenomena with results that ought to be of merit, but we are reluctant to cite all of these here.
However, the sheath in plasmas gets modified depending on the relative plasma constituents and, as a result, ion Mach number and dust critical Mach number have been shown to decline [22]. Several observations have been made in different plasma constituents, e.g., plasma with Boltzmannian ions [29], nonisothermal ions [30], or thermal plasmas [31, 32]. Further studies have been conducted of complex plasma contaminated with negatively charged dust grains. We assume that the temperature of the plasma constituents increases as number of charges residing on the dust grains. We assume that the temperature of the plasma constituents increases as number of charges residing on the dust grains. Thus, the charge neutrality condition at equilibrium, and the dynamical behaviours of levitated dust grains in sheaths could lead to stable dust clouds in the sheaths and yield the basis of nebulon formation in space [35, 36].

The present work seeks to study sheath formation in dusty plasmas coexisting with adiabatically heated ions and electrons, with a view to determining their role in sheath formation. The paper has been organized as follows:

Section 2 describes the basic equations governing the plasma dynamics, and thereafter a sheath equation is derived that is conditioned on the Bohm criterion along with a limit on the critical velocity, which might play a vital role in sheath existence. In section 3, the sheath equation is solved numerically by Runge–Kutta (R–K) method to exhibit the sheath properties for some typical plasma parameters. Overall conclusions are summarized in section 4.

2. Basic equations and derivation of sheath equation

We consider unmagnetised dusty plasma coexisting with adiabatically heated electrons and ions. The partial presence of electrons shows some noble features on nonlinear waves which cannot be explained by considering that all the electrons are absorbed to charge the dust grains. Thus, \( n_{d0} = n_{e0} + Z_d n_{e0} \) defines the charge neutrality condition at equilibrium, where the suffix zero represents unperturbed quantity and \( Z_d \) is number of charges residing on the dust grains. We assume that the temperature of the plasma constituents increases adiabatically as \( T_j \propto \eta_{d1}^{-1} \) \( j = 1, e \) represents the temperatures for ions and electrons, and \( \gamma = 3 \) is the adiabatic heat constant. Basic equations, following Shchekinov [29], are written in normalized form as

\[
\frac{\partial n_d}{\partial t} + \frac{\partial}{\partial x} (n_d u_d) = 0 \tag{1}
\]

\[
\frac{\partial u_d}{\partial t} + u_d \frac{\partial n_d}{\partial x} = \frac{\partial \phi}{\partial x} \tag{2}
\]

\[
\frac{\partial^2 \phi}{\partial x^2} = n_d + \frac{\chi}{1 - \chi} n_e - \frac{1}{1 - \chi} n_i \tag{3}
\]

\[
\frac{\partial \phi}{\partial x} + \gamma n_j \gamma^{-2} \frac{\partial n_j}{\partial x} = 0; \text{ where } j = i, e. \tag{4}
\]

Here, \( n_d \) is the mass of the dust grains moving with velocity \( u_d \) normalized by dust acoustic speed \( C_d = (Z_d kT_d/m_d)^{1/2} \). Potential \( \phi \) has been normalized by \( kT_j/e \). Time and space variables are normalized respectively by the inverse dust plasma frequency \( \omega_{pd} = (n_{d0} Z_d e^2 / \epsilon_0 m_d)^{1/2} \) and Debye length \( \lambda_{pd} = (\epsilon_0 kT_j/m_d)^{1/2} \). Equation (4) defines the motion of adiabatically heated electrons and ions supported by the relation \( P_j \propto n_j^2 \), where \( P_j \) is the pressure. The Poisson equation (equation (3)) has been simplified with the use of charge neutrality condition as

\[
\frac{n_e n_{d0}}{n_{d0}} + \chi = 1 \text{ where } \chi = \frac{n_{e0}}{n_{d0}}.
\]

Now, in deriving the Sagdeev potential equation, the pseudopotential method allows us to assume that the plasma parameters vary as functions of \( \xi = x - M t \) with respect to a frame moving with velocity \( M \) (defined as Mach number).

The basic equations are then modified to the following form:

\[
-M \frac{\partial n_d}{\partial \xi} + \frac{\partial}{\partial \xi} (n_d u_d) = 0 \tag{5}
\]

\[
-M \frac{\partial u_d}{\partial \xi} + u_d \frac{\partial n_d}{\partial \xi} = \frac{\partial \phi}{\partial \xi} \tag{6}
\]

\[
\frac{\partial^2 \phi}{\partial \xi^2} = n_d + \frac{\chi}{1 - \chi} n_e - \frac{1}{1 - \chi} n_i \tag{7}
\]

\[
\frac{\partial \phi}{\partial \xi} + \gamma n_j \gamma^{-2} \frac{\partial n_j}{\partial \xi} = 0 \tag{8}
\]

from which densities are derived as

\[
n_d = \frac{M}{\sqrt{M^2 + 2\phi}} \tag{9}
\]

and

\[
n_j = \left[ 1 \pm \frac{\gamma - 1}{\gamma} \beta \Phi \right]^{\gamma^{-1}} \text{ where } \beta = \frac{T_i}{T_j} \tag{10}
\]

where \( \beta = 1 \) for ions and \( \beta = \frac{e}{k} \) for electrons.

Now the Poisson equation, with boundary conditions as \( \Phi \to 0, \Phi_i \to 0, \xi \to \pm \infty \), derives the Sagdeev equation as

\[
\frac{1}{2} \left( \frac{\partial \phi}{\partial \xi} \right)^2 + V(\Phi, M) = 0 \tag{11}
\]

where

\[
V(\Phi, M) = M^2 \left[ 1 - \left( 1 + \frac{2 \Phi}{M^2} \right)^{1/2} \right] + \frac{\chi}{(1 - \chi) \beta} \times \left[ 1 - \left( 1 + \frac{\gamma - 1}{\gamma} \beta \Phi \right)^{\gamma^{-1}} \right] + \frac{1}{1 - \chi} \left[ 1 - \left( 1 - \frac{\gamma - 1}{\gamma} \Phi \right)^{\gamma^{-1}} \right]. \tag{12}
\]
Equation (11) is the desired sheath equation and variation $V(\Phi, M)$ yields the nature of sheath formation in plasma. Next, the Bohm sheath criterion, based on $V(\Phi, M) < 0$, yields

$$\left(\frac{\partial^2 V}{\partial \Phi^2}\right)_{\Phi=0} < 0$$

(13)

from which Mach number derives

$$M > \sqrt{\frac{(1 - \chi)\gamma}{1 + \chi\beta}}.$$  

(14)

The Bohm sheath criterion exhibits the effect of heated electrons and ions. It is obvious that $M = \sqrt{\gamma}(\approx 1.74$ for $\gamma = 3$, $\chi = 0$ (i.e. the case of complete electron depletion) evaluates the lower limit. Again, $V(\Phi, M) > 0$ yields a condition to $\chi_{\text{crit}}$ beyond which the Sagdeev potential equation (11) takes the form

$$\frac{\chi_{\text{crit}}}{\beta} \left\{ 1 - \left(1 - \frac{\beta}{3} M^2\right)^{3/2} \right\} + \left(1 - \chi_{\text{crit}}\right)M^2$$

$$+ \left\{ 1 - \left(1 + \frac{M^2}{3}\right)^{3/2} \right\} = 0.$$

(15)

Equation (11) has been solved numerically by fourth-order Runge–Kutta method to exhibit the salient features of the sheath. These initial value problems need to calculate the wall potential, and have been done by balancing the flux of charged dust grains and electrons with the ion flux at the boundary wall (discussed in the next section).

3. Numerical results

Equation (11) is the ideal equation for studying the existence and salient features of a sheath in plasma. To find the nature of the sheath, we apply energy conservation at the wall to determine wall potential. In the present plasma model, adiabatically heated electrons are lost on the wall more rapidly before the charged dust grains. This might leave the wall with a net positive or negative potential depending on the density ratio, $\chi$. The wall potential adjusts self-consistently and fluxes of ions, electrons, and charged dust grains are balanced through the charge neutrality condition, which enables us to evaluate the wall potential $\Phi_w$ as

$$(1 - \chi) + \chi \mu \left(1 + \frac{\gamma - 1}{\gamma} \frac{\Phi_w}{C_d}\right)^{1/\gamma - 1}$$

$$= \left[1 - \frac{\gamma - 1}{\gamma} \Phi_w\right]^{1/\gamma - 1}$$

(16)

with $\mu = \sqrt{m_i/m_e}$, where $m_j$ is the mass of $j$th species. $l = v_i/C_d$, which is the ion velocity normalized with dust acoustic speed and thereby is a function of Mach number, $M$. Since the ions and electrons are hot species, they bombard the wall simultaneously and velocities are assigned with some arbitrary constant value. Here $l$, for ions, has been chosen as $l = 10$. Wall potential $\Phi_w$ is a function of $\chi$ and $\beta$, and its variation is shown in figure 1. For prescribed value $\beta = 0.1$, the wall potential is found to decrease with the increase in density, $\chi$. For $\chi < 0.02$, wall potential is found to be positive while, with increase in electron number density (i.e. for $\chi > 0.02$), more electrons strike the wall rapidly in comparison to ions and render negative potential to the wall. Figure 2 shows that the critical value for $\chi$ equals 0.019. Correspondingly, Mach number becomes high enough that the dust grains become highly energetic and spill off the Debye shielding region, and the wall will be negatively charged. Further, it is shown that $\chi_{\text{crit}}$ attains a negative value for $M \approx 4.43$. This suggests that the dust Mach number cannot exceed the critical Mach number and requires that $M \leq 4.43$ for every $\chi$. This condition sets an upper limit to the dust entering velocity.

After the estimation of wall potential, the sheath equation (equation (11)) has been solved numerically to exhibit the variation of plasma potential (figures 3, 4) with the variation of density $\chi$ and Mach number $M$. For positive wall potential (when $\chi < \chi_{\text{crit}} \approx 0.02$), features of sheath will be well

![Figure 1. Variation of wall potential with $\chi$.](image1)

![Figure 2. Variation of critical density $\chi_{\text{crit}}$ with Mach number $M$.](image2)
defined (figure 3). Figure 3 shows that wall potential (positive) decreases with increase of $\chi$ from 0.001 to 0.01. This clearly implies that, as the relative density of electrons hitting the wall increases, the net positive potential of the wall decreases. However, with an increase in dust fluid velocity $M$ (shown in figure 4), it is capable of entering the sheath despite ion and electron bombardment on the wall, and helps shield the wall potential more effectively; thereby, contraction of sheath thickness occurs. There is a minimum Mach number below which no sheath is formed. For $\chi = 0.01$ and $\beta = 0.1$, the Mach number limit has been calculated as $M > 1.72$. For $M = 1.6$, the potential profile is found to diverge from its usual characteristic behaviour in forming the sheath region, and thus the violation of the Bohm criterion depicts the non-existence of sheath (line marked by I in figure 4).

The preceding discussions have been supported by plotting the variation of potential $V(\Phi)$ with $\Phi$ (figure 5), which shows that the region of the sheath as well as the existence of compressive solitary waves depends on the Mach numbers. For $M_1 = 1.72$ (calculated for $\beta = 0.1$, $\chi = 0.01$), sheath is not shown. However, beyond $M_1 = 1.72$, a prominent sheath as well as rarefactive solitary wave could be found.

Our study finds the existence of compressive solitary waves with the variation of amplitudes and sheath for some typical plasma compositions. It also evaluates successfully the Bohm criterion depending on $\chi$: showing the minimum speed limit of the dust. It calculates the critical plasma density and sets an upper limit to dust Mach number beyond which sheath existence is not possible. Though we have shown the observations with the input of some reasonable numeric values for plasma constituents, changes of input data will give only a schematic variation on graphical presentation and could follow the parallel discussions. Thus, the data input gives an understanding of an advanced theoretical knowledge and could be advantageous in furthering the observations in space and astro-plasmas.

4. Conclusions

Based on pseudopotential method, sheath has been studied rigorously in an adiabatically heated plasma coexisting with charged dust grains. To derive sheath formation, Bohm sheath criterion has been evaluated which yields a limit on Mach
number. The trapping of fewer electrons and ions in plasma potential effectively causes a failure to dissipate heat to plasma constituents. Finally, adiabatically heated plasma affects the plasma potential at the boundary ($\Phi_0$) and sets a limit to the dust fluid velocity ($M$). Further, given the presence of dust in the dynamical system, sheath formation is affected by the variation of critical density $\chi_{\text{crit}}$. For $\chi < 0.02$, sheath does not yield in plasma. Again critical density $\chi_{\text{crit}}$ varies with Mach number and becomes negative for $\chi_{\text{crit}} > 0.019$ which, in turn, sets an upper limit to the Mach number. Schematic variations in sheath structures have been shown for three different cases depending on density $\chi$ and dust fluid velocity $M$. It has been shown that the existence of sheath formation depends on the limit for a certain allowable range in $\chi$ as well as $M$. This has been verified with Sagdeev potential $V(\Phi)$ variation, and it is found that sheath in plasmas can be evaluated beyond the critical lower limit of density and allowable range of Mach numbers. As a whole, we conclude that adiabatically heated electrons and ions have an important role in the execution of sheath width at the plasma boundary. The characteristic behaviors of robust sheath for some input typical plasma constituents are shown explicitly.

The totality of our investigation shows the effect of adiabatically heated plasma and dust charging on narrowing the sheath width. Now, we combine observations of sheath over the Earth’s moon [33] and the dynamical behaviors of levitated dust grains into the sheath [23, 31], which has been studied recently by us and other authors. It is shown that the dust grain levitation and its oscillations are trapped within the sheath, and this leads to the formation of dust clustering similar to the features of nebulon formation.

Actually, when a single dust grain enters into the sheath region, the objective dust is charged highly and generates forces on it that cause the dust grain either to sink, float, or be stable within the robust sheath. Several dust grains, with varying dust sizes, will be stable in different places centering around the heavier grains, with a sequential arrangement as to size. It has been seen that the denser dust grains will be agglomerated almost in the middle of sheath, surrounded by thinner-sized dust grains. In this process, levitated dust grains allow for the formation of stable dust clouds in the sheath zone. Adiabatically heated electrons and ions play a significant role in the formation of sheath and in the shrinking of it as well. Due to the contraction of the sheath region, numerous dust grains will be stable and form nebulae. Linking with earlier works on the dynamical behaviors of levitated dust grains in sheath, we believe that the dust-grain cloud is made to compress onto itself, increasing gravitational attraction, which in turn causes the sheath to shrink further, agglomerating more and more heavy dust grains inside the sheath. The shrinking of sheath thickness allows for continuous compression and forms a chunk of well-defined dust atmosphere similar to nebulae, and could be observed in every regions of the universe. It is true that the nebula always contracts under the influence of adiabatic effect and gravity. Again, their role allows the dust cloud to move fast with the contraction of the sheath, causing increasing rotation which, in turn, causes heavy dust grains to bulge at its centre.

The observed phenomena could be of interest in astrophasmas and there must be further advanced study to find the sheath properties. After attaining a robust sheath, the dynamical behaviors of dust grains should be studied with the aim of finding the basis of nebulon formation, which in turn requires further research into the atmospheres of moons, asteroids, slow and high rotating stars, pulsars, and magnetospheres. The present study could be advantageous to rekindling headway in the study of astrophysical problems. The investigation has not been taken up yet, but we are sure that such study is warranted given the potential of many more new findings on the dynamical behaviors of dust grains in sheath as the basis of nebulons in space and astrophasmas.

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References

[27] Chen X P 1998 Phys. Plasmas 5 804