Directional power absorption in helicon plasma sources excited by a half-helix antenna

Mohsen AFSHARMANESH and Morteza HABIBI

Department of Energy Engineering and Physics, Amirkabir University of Technology, Tehran, Iran
E-mail: mortezahabibi@aut.ac.ir

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Abstract
This paper deals with the investigation of the power absorption in helicon plasma excited through a half-helix antenna driven at 13.56 MHz. The simulations were carried out by means of a code, HELIC. They were carried out by taking into account different inhomogeneous radial density profiles and for a wide range of plasma densities, from $10^{11}$ cm$^{-3}$ to $10^{13}$ cm$^{-3}$. The magnetic field was 200, 400, 600 and 1000 G. A three-parameter function was used for generating various density profiles with different volume gradients, edge gradients and density widths. The density profile had a large effect on the efficient Trivelpiece–Gould (TG) and helicon mode excitation and antenna coupling to the plasma. The fraction of power deposition via the TG mode was extremely dependent on the plasma density near the plasma boundary. Interestingly, the obtained efficient parallel helicon wavelength was close to the anticipated value for Gaussian radial density profile. Power deposition was considerably asymmetric when the $n_B$ ratio was more than a specified value for a determined density width. The longitudinal power absorption was symmetric at approximately $n_B = 10^{11}$ cm$^{-3}$, irrespective of the magnetic field supposed. The asymmetry became more pronounced when the plasma density was $10^{12}$ cm$^{-3}$. The ratio of density width to the magnetic field was an important parameter in the power coupling. At high magnetic fields, the maximum of the power absorption was reached at higher plasma density widths. There was at least one combination of the plasma density, magnetic field and density width for which the RF power deposition at both side of the tube reached its maximum value.

Keywords: helicon plasma, half-helix antenna, asymmetric power absorption, density profile, HELIC code

(Some figures may appear in colour only in the online journal)

1. Introduction

Helicon plasma is known for its markedly high ionization efficiency. Plasma densities up to $10^{19}$ m$^{-3}$ can be obtained, using magnetic fields of less than 1000 G [1]. One promising way to increase the plasma density downstream of the tube for plasma application is to use a type of antenna that deposits RF power asymmetrically. With regard to the excitation of different azimuthal mode number $m$ in helicon plasmas, various types of antennas have been constructed. A single-loop antenna is azimuthally symmetric and couples to the $m = 0$ helicon mode. The double-saddle and Nagoya-III antennas are constructed to launch plane polarized waves that are superpositions of the $m = \pm 1$ modes, while helical antennas excite either the $m = +1$ or $m = -1$ modes, depending on their helicity. Helicon discharge exhibits an axial symmetry structure for the Nagoya-III antenna. However, it is profoundly asymmetric when RF power is transmitted to the plasma by helical antennas [2–4]. In [5], in order to achieve higher plasma density, the Nagoya-III antenna was replaced by a helical antenna; ten times higher extracted ion beam current was expected by means of a helical antenna. As a result, they used a helical antenna in their subsequent work [6]. The propagation and damping properties of the $m = \pm 1$ modes have been studied in many theoretical and
experimental works [7–14]. The value of the density gradient is an important parameter in the propagation of the \( m = -1 \) mode [2, 4, 7, 13]. It displays a cut-off depending on the profile width and the plasma parameters [2, 8]. For a specific width, this mode is diminished if the wave frequency and the ratio of the plasma density to the magnetic fields are more than a critical value (i.e. at a magnetic field of 1000 G, plasma densities less than \( 1.5 \times 10^{19} \text{m}^{-3} \) and density width of 1.5 cm [8]). One of the important parameters in the helicon simulations is the plasma density profile. Although the Bessel or parabola radial density profile are often supposed in many works, the Gaussian radial density profile is closer to the experimental observations [13].

Investigation of the power deposition is one of the key research topics in helicon plasmas. Since the wave damping and power absorption cannot be explained by collisions and Landau damping, Trivelpiece–Gould (TG) modes have been proposed as another mechanism. TG modes are slow waves that can be easily absorbed and heat electrons [15]. The TG mode can be excited because of surface conversion or bulk mode conversion [16]. Different codes such as HELIC [17–19], ANTENA2 [20], SPIREs [21] and ADAMANT [22] have been developed for power absorption estimation. All of these codes obtain the electromagnetic field, power absorption and antenna spectrum of different antenna types. However, the simulation approaches and the power absorption mechanisms that have been considered are quite different. ANTENA2 code is based on the both collisional and Landau damping heating mechanisms. Similar to ANTENA2, in HELIC and SPIREs each antenna is specified by its Fourier transform. In contrast to ANTENNA2 code, in HELIC and SPIREs the power absorption is calculated considering collisional plasma. The finite-difference frequency-domain method is employed in SPIREs, while HELIC uses the numerical integration method. ADAMANT solves a system of volume and surface integral equations for solving the problem. In this work, the power deposition for both helicon and TG modes are investigated. The essential physical conditions that are necessary for asymmetric power deposition are presented. The simulations were carried out with HELIC code [17–19] for different plasma density profiles.

2. Basic theory and description of HELIC code

HELIC [17–19] is a C++ program for simulating RF wave excitation and propagation in helicon and ICP plasma sources. It calculates the power deposition for different conventional antenna types. Power absorption of both helicon and TG modes are supposed in HELIC. Radial plasma density could be uniform, polynomial and also a three-parameter function as follows:

\[
\frac{n}{n_0} = \left[ 1 - \left( \frac{r}{w} \right)^s \right]^t, \quad w = \frac{a}{\left[ 1 - h_a r^{1/t} \right]^{1/s}}
\]

(1)

where \( s \) and \( t \) are constant parameters. Here \( h_a \) is the relative density \( n/n_0 \) at the plasma edge and \( n_0 \) is the plasma density at the center. If \( h_a \) is set to a non-zero value, a vast range of profiles from nearly flat to extremely peaked on axis can be fitted. Maxwell’s equations in conjunction with the usual cold-plasma dielectric elements are used to create a system of differential equations. The permittivity tensor reads as:

\[
\varepsilon = \begin{bmatrix}
\varepsilon_{xx} + i \varepsilon_{xy} & 0 \\
-i \varepsilon_{xy} & \varepsilon_{xx} & 0 \\
0 & 0 & \varepsilon_{zz}
\end{bmatrix}
\]

(2)

where the tensor elements are the Stix parameters [23]:

\[
\varepsilon_{xx} = 1 - \frac{\omega_{pe}^2}{\omega^2 + i \omega \gamma_e} \quad (3a)
\]

\[
\varepsilon_{xy} = -\frac{\omega_{pe}^2}{\omega (\omega + i \gamma_e)^2 - \Omega_e^2} \quad (3b)
\]

\[
\varepsilon_{zz} = 1 - \frac{\omega_{pe}^2}{\omega (\omega + i \gamma_e)} \quad (3c)
\]

where \( \omega_{pe} \) is the plasma frequency, \( \Omega_e \) is gyro frequency, \( \omega \) is the wave frequency (3.56 MHz) and \( \gamma_e \) is the effective collisional frequency. In this code, plasma discharge is not studied, rather this code simulates plasma response and computes resistivity as a function of magnetic field for a specific plasma density. The harmonic waves have been supposed to vary as \( \text{exp}\{i(m \theta + k_z - \omega t)\} \). The \( z \) axis is aligned with the magnetic field. The code solves the following coupled radial differential equations for each \( k_z \):

\[
\frac{\partial E_r}{\partial r} = -i m \frac{E_r}{r} + \frac{E_z}{r} + \frac{1}{c^2} \frac{\partial B_z}{\partial t} \quad (4a)
\]

\[
\frac{\partial E_z}{\partial r} = -i k E_r - \frac{1}{c^2} \frac{\partial E_z}{\partial t} \quad (4b)
\]

\[
\frac{\partial B_r}{\partial r} = \frac{m}{r} \frac{k}{\omega} E_z - \frac{1}{c^2} \frac{\partial E_r}{\partial t} + \left( \frac{\varepsilon_{xx} - \frac{m^2}{k_0^2 r^2}}{\varepsilon_{xx} + 1} \right) \frac{\omega}{c^2} E_z \quad (4c)
\]

\[
\frac{\partial B_z}{\partial r} = -\frac{1}{c^2} i \frac{k}{\omega} E_r + (k^2 - k_0^2 \varepsilon_{xx}) \frac{E_z}{\omega} + \frac{m}{r} E_z \quad (4d)
\]

Here, \( m \) is the azimuthal mode number, \( E \) and \( B \) are the electric and magnetic fields in cylindrical coordinates, respectively, \( k \) is the axial wavenumber, \( k_0 = \frac{\omega}{c} \) and \( \omega \) is the wave frequency. In HELIC, common types of antennas can be expressed by their Fourier transform. The antenna wires are assumed to lie on a cylinder of radius \( b \), and a conducting wall of radius \( c \) surrounds the system, as illustrated in figure 1.

The antenna is presumed to be an infinitely thin sheet at \( r = b \) having a sheet current \( J \) with \( \nabla \cdot J = 0 \). We must only determine the azimuthal current density since the axial current density can be calculated from the \( m = -J \) transform of the continuity equation. If the antenna current is a surface current \( J(r, \phi) \) on the surface \( r = b \), it is expressed in terms of the Fourier components \( J(m, k) \) by:

\[
J(\phi, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk \sum_{m=-\infty}^{\infty} J(m, k) e^{i(m\phi + kz)} \quad (5)
\]

where:

\[
J(m, k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dz \int_{2\pi}^{0} d\phi J(\phi, z) e^{-i(m\phi + kz)} \quad (6)
\]
For half-helix antennas, that is
\[ J_c(m, k) = -\frac{L_0 k L}{\pi m} \sin \left( \frac{(kL - mn\pi)}{2} \right) \],
the power transmitted to the antenna is
\[ P_{\text{ant}}(k) = \frac{1}{2} \text{Re} \left( \int E^* \cdot J_{\text{antenna}} d^2 r \right). \]
The absorbed power depends on the plasma loading resistance and it can be expressed as:
\[ P_{\text{abs}} = -\frac{1}{2} R_p J_{\text{ant}}^2. \]
The longitudinal power deposition profile \( P_z(z) \) is specified as the absorbed power between \( z \) and \( z + dz \) integrated over the cross section of the plasma column. The radial absorption profile \( P_r(r) \) is expressed as the power absorbed in a narrow shell along the plasma tube. A more detailed description of the code can be found in [17–19].

3. Simulations
A cylindrical discharge, with an outer tube radius of 2 cm and length of 30 cm, with a half-helix antenna of 2.5 cm radius, launching at \( f = 13.56 \text{ MHz} \), has been considered (figure 2). Results were obtained for the magnet field of \((B_0) 200, 400, 600 \) and \( 1000 \text{ G} \). Simulations were done at a pressure of 10 mTorr \( \text{Ar} \) gas and at \( n_0 = 10^{17} \) to \( 10^{19} \text{ m}^{-3} \) plasma density. For more clarity, the axial power profile for different \( z \) values, the radial power profile for different \( r \) values and the curves of total absorbed power at both sides of the antenna are examined. The total power absorption is calculated by integrating \( P_z(z) \) along the plasma tube:
\[ P = 2 \int_{-a}^{a} P_z(z) dz. \] The antenna center is placed at \( Z = 0 \).

The antenna is supposed to have a helicity that initiates \( m = +1 \) helicon modes on the \(+z\) side and \( m = -1 \) modes on the \(-z\) side. The plasma is considered uniform in temperature \((T_e = 3 \text{ eV})\). Calculations were carried out for different radial density profiles. The density profiles were parameterized as equation (1). We examined the effect of the plasma density profile on the spatial power absorption by changing one of the \((s, t, h_0)\) parameters and fixing the others.

4. Result and discussion
According to the above explanations, different radial plasma density profiles were employed for the power absorption calculations. The results are outlined as follows. We first examined the effect of the radial plasma density profile on the efficient antenna coupling dealing with antenna coupling to the helicon and TG modes. In the following parts, we concentrated on the effects of the plasma density, density width and magnetic field on the power absorption.

4.1. Volume gradient
First, we examined the effect of volume gradient on the power absorption. Calculations were carried out for four different density profiles by changing the \( t \) parameter in equation (1). For more clarity, the effect of the volume gradient on the radial power absorption was also considered. The structure of \((s, t, h_0) = (2, t, 0.001)\) was used as reference. The parameters were: \( B_0 = 400 \text{ G} \) and \( n_0 = 10^{17} \text{ cm}^{-3} \). The density profiles and the corresponding power absorption are shown in figure 3. The parallel helicon wavelength for a uniform plasma densities is given by [24]:
\[ \lambda_p = \frac{3.83 B_0}{ae \mu_0 f} \]
where \( B_0 \) is in Tesla, \( n \) in m\(^{-3} \) and \( f \) in Hz. The anticipated parallel helion wavelength for half-helix antenna is \( \lambda_p = 2l_{\text{ant}} \). The antenna length is given by:
\[ l_{\text{ant}} = \sqrt{d^2 + (\pi a)^2} \]
where \( d \) is the distance between the end rings and \( a \) is the radius of the tube. In our work, \( d = 10 \text{ cm} \) and \( a = 2 \text{ cm} \). Thus we have:
\[ l_{\text{ant}} \approx 11.8 \text{ cm} \implies \lambda_p = 2l_{\text{ant}} \approx 23.61 \text{ cm} \]
Antenna lengths of the order of $l = \frac{\lambda}{2}$ were found to provide the best energy coupling [25]. We found that there was a discrepancy between the expected and simulated value of the optimum antenna length and the helicon wavelength. As is clear, when $t = 1$ the optimum antenna length is 13 cm and $\lambda_0 = 26$ cm. The efficient antenna length increases with $t$; it is 19 cm when $t = 6$. From figure 4 it can be deduced that the helicon mode is the dominant power absorption mechanism in comparison to the TG mode. For narrow density profile (i.e. $t = 60$) the fraction of the power absorption at the plasma volume increases for $l_{\text{ant}} = 19$ cm.

4.2. Edge gradient

Second, we examined the effect of the edge gradient on the power coupling to the plasma. The structure of $(s, t, h_0) = (2, t, 0.001)$ is used as a reference. The parameters were: $B_0 = 400$ G and $n_0 = 10^{13}$ cm$^{-3}$. By varying the $s$ parameter, different density profiles were achieved. As depicted in figure 5, efficient coupling to the plasma occurs at shorter antenna lengths at higher values of $s$. According to the radial power absorption profile in figure 6, the helicon mode is again the dominant heating mechanism in comparison to the TG mode. When $s = 1$ the optimum antenna length is 19 cm and $\lambda_0 = 38$ cm. The efficient antenna length decreases with $s$; it is 11 cm when $s$ is equal to 10.

4.3. Density width

Next, we examined the effect of density width on the power absorption. The configuration of $(s, t, h_0) = (2, 10, h_0)$ was used as reference. Parameters are: $B_0 = 400$ G and $n_0 = 10^{13}$ cm$^{-3}$. By changing $h_0$, different density profiles were obtained. The $(s, t) = (2, 10)$ profile is close to a Gaussian and is near to the experimental observations. Thus, we will use this profile in our next calculations. As evidenced

![Figure 3](image1.png)

**Figure 3.** Effect of the volume gradient on the power absorption for different antenna lengths. (a) Density profiles, (b) power absorption. The structure of $(s, t, h_0) = (2, t, 0.001)$ is used as a reference. Other parameters are: $B_0 = 400$ G and $n_0 = 10^{13}$ cm$^{-3}$.

![Figure 4](image2.png)

**Figure 4.** Radial power absorption for $l_{\text{ant}} = 19$ cm.

by the curves plotted in figure 7, the optimum antenna length is 11 cm, irrespective of the value of the density width considered. The antenna length was also 11 cm in the next simulations. The interesting result is that the corresponding efficient parallel helicon wavelength is \( \lambda_p = 22 \) cm, which is close to the predicted value. The main feature of the power absorption for this density profile is the dominance of the TG mode in comparison to the helicon mode. It is seen from figure 8 that most of the RF power is deposited at the plasma edge. The experimental results indicate that the parallel wavelength is observed to vary in a discrete manner in accordance with the system length \[26\]. In other words, the wavenumber \( k_p \) is given by:

\[
k_p = \frac{2\pi}{\lambda_p} \quad \Rightarrow \quad \lambda_p = \frac{2L}{n} = 60 \text{ cm}
\]

where \( L \) is the system length and \( n \) is an integer (helicon mode number). A typical power absorption spectrum for the structure of \((s, t, h_0) = (2, 10, 0.3)\) at \( B_0 = 400 \) G and \( n_0 = 10^{13} \text{ cm}^{-3} \) is depicted in figure 9. The \( n = 2 \) helicon mode has the most important contribution to the power absorption. By substituting in equation \((10)\), the parallel wavelength and the corresponding wavenumbers are \( \lambda_p = 0.2810 \) and \( k_p = 22.38 \text{ m}^{-1} \), respectively. This is close to the wavenumber of the \( n = 2 \) helicon mode. In agreement with \[25\] the discrepancy between the anticipated and calculated helicon wavelength can be attributed to the plasma density profile. Figures 10 and 11 show the total power absorption and the power absorption at the positive and negative sides of the antenna for the \((s, t, h_0) = (2, 10, 0.3)\) configuration at 200 and 1000 G, respectively. We find that at 200 and 1000 G, \( P(z) \) is profoundly symmetric at plasma densities of less than \( 4 \times 10^{11} \) and \( 4 \times 10^{12} \text{ cm}^{-3} \), respectively. It seems that the

Figure 5. Effect of the edge gradient on the power absorption for different antenna lengths. (a) Plasma density, (b) power absorption. The structure of \((s, t, h_0) = (s, 10, 0.001)\) is used as reference. The parameters are: \( B_0 = 400 \text{ G} \) and \( n_0 = 10^{13} \text{ cm}^{-3} \).

Figure 6. Radial power absorption for \( l_{ant} = 24 \text{ cm} \).
Asymmetric axial power deposition appears when the $\frac{n}{B_0}$ ratio is more than a specific value. This ratio is equal to $2 \times 10^9$ and $4 \times 10^9$ cm$^3$ G$^{-1}$, respectively. This is in agreement with the helicon dispersion relation, which means that at lower $\frac{n}{B_0}$ helicon modes become evanescent. In general, there is at least one set of plasma density and magnetic field for which power absorption is a maximum. The longitudinal power absorption is symmetric at $n = 10^3$ cm$^{-3}$ approximately independent of the magnetic field. The asymmetry becomes more pronounced when the plasma density is $10^{12}$ cm$^{-3}$. This is due to the propagation of the helicon mode needing a plasma density more than a critical value (i.e. $10^{11}$ cm$^{-3}$). In agreement with [22] power absorption reaches a maximum value for different magnetic fields, by changing the plasma density. In the capacitive coupling regime the wave electric field can penetrate into the plasma and power absorption increases linearly with $n_e$. In the inductive coupling

Figure 7. Effect of the density width on the power absorption for different antenna lengths. (a) Plasma density, (b) power absorption. The configuration of $(s, t, h_a) = (2, 10, h_a)$ is used as reference. The parameters are: $B_0 = 400$ G and $n_0 = 10^{13}$ cm$^{-3}$.

Figure 8. Radial power absorption for $l_{ant} = 11$ cm.

Figure 9. Power absorption versus parallel wavenumber for $l_{ant} = 20$ cm for the $(s, t, h_a) = (2, 10, 0.3)$ density profile. Other parameters are: $B_0 = 400$ G and $n_0 = 10^{13}$ cm$^{-3}$.
regime it is quite different; the skin depth is smaller than the system dimensions. Thus, a fraction of the wave cannot penetrate into the plasma. In this regime, the power absorption reaches a maximum and then decays with \( n_e^{1/2} \) at high densities [27]. In the following, the influence of the profile width on the longitudinal power deposition is studied by changing \( h_a \) and fixing \( s \) and \( t \) to 2 and 10. As shown in figure 12, the power absorption increases with \( h_a \). The maximum of the power absorption occurs at \( h_a = 0.3 \) at 200 G. But, at 1000 G the power absorption is a strictly increasing function of \( h_a \). It seems that the ratio of \( h_a/B_0 \) is an important parameter in the energy coupling. Figures 13 and 14 depict \( P_z(z) \) at the middle of the plasma column \((z = 0)\) and at both side of the plasma column \((z = \pm 15 \text{ cm})\) for different magnetic fields. The axial power absorption at the center of the tube increases with \( h_a \). At both sides of the plasma column it is quite different; the variation of the power absorption with \( h_a \) is different at various magnetic fields. In contrast to 200 G, at 600 and 1000 G the maximum of the power absorption occurs at higher values of \( h_a \). It seems that at high magnetic fields, the maximum of the power absorption is reached at high plasma density widths. An outline of the result is

**Figure 10.** Power absorption at \( B_0 = 200 \text{ G} \) for the \((s, t, h_a) = (2, 10, 0.3)\) plasma density structure.

**Figure 11.** Power absorption at \( B_0 = 1000 \text{ G} \) for the \((s, t, h_a) = (2, 10, 0.3)\) plasma density structure.

**Figure 12.** Total power absorption at \( n_0 = 6 \times 10^{12} \text{ cm}^{-3} \) for different \( h_a \), and \( B_0 = 200 \) and 1000 G.

**Figure 13.** Power deposition dependency on \( h_a \) at the center \((z = 0)\) at \( n_0 = 6 \times 10^{12} \text{ cm}^{-3} \) and different magnetic fields for the \((s, t, h_a) = (2, 10, h_a)\) density profile.
that there is a certain value of the $\frac{h_a}{B_0}$ ratio for which the power absorption at both sides of the antenna is a maximum.

A typical asymmetric axial power absorption profile at $h_a = 0.3$ is shown in figure 15. The maximum of the power deposition is located inside the antenna and on its positive side.

5. Conclusion

Power absorption was computed using the HELIC code for different radial density profiles. The collisional power absorption due to both helicon and TG were calculated. Different plasma density profiles were generated by using a three-parameter function. The radial density profile had a profound effect on the helicon and TG excitation, and antenna coupling to the plasma. The calculated efficient parallel helicon wavelength was $\lambda_l = 22$ cm, which was close to the predicted value for a Gaussian radial density profile. For other radial density profiles, the discrepancy between the anticipated and calculated helicon wavelengths could be attributed to the structure of the plasma density profile. We found that at 200 and 1000 G the axial power absorption was profoundly symmetric at plasma densities of less than $4 \times 10^{11}$ and $4 \times 10^{12}$ cm$^{-3}$, respectively. It seems that the asymmetric axial power deposition appears when the $\frac{n}{B_0}$ ratio is more than a specific value. This ratio was equal to $2 \times 10^9$ and $4 \times 10^9$ cm$^3$ G$^{-1}$ at 200 and 1000 G, respectively. The longitudinal power absorption was symmetric at low densities, i.e. $n_0 = 10^{11}$ cm$^{-3}$; approximately independent of the magnetic field. In general, there was at least one set of plasma density and magnetic field for which power absorption was a maximum. The maximum of the power absorption occurred at $h_a = 0.3$ (relative density at edge) at 200 G. However, at 1000 G the power absorption was a strictly increasing function of $h_a$ for a Gaussian density profile. The ratio of $\frac{h_a}{B_0}$ is an important parameter in the axial power absorption. Axial
power absorption at the center of the tube increased with $h_a$. In contrast to 200 G, at 600 and 1000 G the maximum of the power absorption at $z = \pm 15$ cm occurred at higher values of $h_a$. There was a certain value of the $\frac{h_a}{h_0}$ ratio for which the power absorption at both side of the antenna was a maximum. The parallel wavelengths varied in a discrete manner in accordance with the plasma tube size. The power absorption spectrum reached its maximum value at a parallel wave-number, which was close to the helicon dispersion relation. Due to the asymmetric power deposition in the half-helix antenna, it is a good choice for plasma application especially for our future work on ion extraction.

References