Effect of Fuelling Depth on the Fusion Performance and Particle Confinement of a Fusion Reactor

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Abstract  The fusion performance and particle confinement of an international thermonuclear experimental reactor (ITER)-like fusion device have been modeled by numerically solving the energy transport equation and the particle transport equation. The effect of fuelling depth has been investigated. The plasma is primarily heated by the fusion produced alpha particles and the loss process of particles and energy in the scrape-off layer has been taken into account. To study the effect of fuelling depth on fusion performance, the ITERH-98P(y,2) scaling law has been used to evaluate the transport coefficients. It is shown that the particle confinement and fusion performance are significantly dependent on the fuelling depth. Deviation of 10% of the minor radius on fuelling depth can make the particle confinement change by $\sim 61\%$ and the fusion performance change by $\sim 108\%$. The enhancement of fusion performance is due to the better particle confinement induced by deeper particle fuelling.

Keywords: particle fuelling, particle confinement, ITER

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(Some figures may appear in colour only in the online journal)

1 Introduction

The International Thermonuclear Experimental Reactor (ITER) will be the first fusion device which could maintain the burning plasma with fusion energy gain factor $Q$ (the ratio between total fusion released power and the input heating power) larger than 10 for considerable duration $[1-7]$. For such a high performance, the device size of ITER should achieve an unprecedented level based on the existing confinement technology. For instance, the major radius of ITER is 6.2 m, which is 109.5% larger than that of the existing largest tokamak Joint European Torus (JET) which has a major radius of 2.96 m $[8]$. With the huge size scale even bigger than before, some physics issues, including the fuelling of particles, would be more complex.

The main fuelling methods of the existing tokamaks are neutral beam injection (NBI), gas puffing (GP) and pellet injection (PI) $[9,10]$. NBI, which is highly efficient in present devices, cannot supply enough particle flux for the 1 MeV neutral beam because of the large size scale of ITER $[9]$. GP and PI will be the primary fuelling methods of ITER. The efficiency of GP will also be much less in ITER since the scrape-off layer (SOL) of ITER will be significantly thicker (and also hotter) than that of the present tokamaks. PI, which is considered as a deep fuelling method today, will be a relatively shallow fuelling means (penetration up to 20%–30% of the minor plasma radius $[9]$) in ITER due to the large device size and high parameters. From the above, it can be seen that all of the main existing fuelling methods will have less efficiency and shallower penetration depth in ITER.

The effect of fuelling depth on particle confinement was studied in Refs. $[11-13]$ with the experimental data on Japan Atomic Energy Research Institute Tokamak-60 Upgrade (JT-60U) $[14]$. However, since the macroscopic parameters of JT-60U are far from ITER, the studies above cannot demonstrate the situations in a fusion reactor with long pulse burning plasma; a detailed study on the effect of fuelling depth on the particle confinement and fusion performance of a fusion reactor is still lacking. In this paper, the temperature and density profiles of an ITER-like fusion device have been modeled in different fuelling depth conditions. To study the effect of particle fuelling depth on fusion performance, the evolution of the temperature profile is coupled with the density profile with the ITERH-98P(y,2) scaling law $[4,5]$: to study the direct effect of fuelling depth on particle confinement, the evolution of density profile is modeled with a set of fixed transport coefficients. The remaining parts of this paper are organized as follows. In section 2, we present the basic theoretical model; in section 3, the numerical results are

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presented and discussed; in section 4, we summarize the main results.

2 Theoretical model

The simulations are carried out by using an earlier developed one-dimensional transport code \[7\], which is extended to include the transport process in the SOL region in this paper. With the assumption that the ion and electron have the same temperature \( T \) and the quasi-neutral condition, the particle and heat transport can be described by the following equations:

\[
\frac{\partial n}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r I_r) + \frac{n}{\tau_\parallel} = S_p, \tag{1}
\]

\[
\frac{\partial}{\partial t} (3nT) + \frac{1}{r} \frac{\partial}{\partial r} (r q_r) + \frac{1}{r} \frac{\partial}{\partial r} (5r I_r T) - \frac{I_r}{n} \frac{\partial}{\partial r} (2nT) + \frac{3nT}{\tau_\parallel} = S_\alpha + S_{aux} - S_{rad}, \tag{2}
\]

\[
I_r = -P_\parallel \frac{\partial n}{\partial r} + nu_r, \tag{3}
\]

\[
q_r = -2n\chi \frac{\partial T}{\partial r}. \tag{4}
\]

where \( n \) is the electron density; \( q_r \) is the radial heat flux due to the thermal conduction; \( I_r \) is the radial particle flux; \( D \) and \( \chi \) are the particle and heat diffusivities, respectively; \( S_p \) is the particle source, \( S_\alpha \) and \( S_{aux} \) are the energy source terms due to the D-T fusion reaction heating and the auxiliary heating, respectively; we assume that the auxiliary heating power density \( (S_{aux}) \) has a centrally peaked radial distribution; \( S_{rad} \) is the energy sink term due to the radiation of plasma; \( u_r \) is the radial particle pinch velocity; \( r \) is the radial coordinate normalized by the plasma minor radius \( a \); \( \tau_\parallel \) is the time-scale of the parallel loss which is induced by the open magnetic force line in the SOL. Note that the endpoint of the computational domain is extended to include the SOL region, the new computational domain ranges from the magnetic axis to the device wall.

\( S_\alpha, S_{aux} \) and \( S_{rad} \) are treated in the same way with the inductive scenario case in Ref. \[7\]. The steady profiles of them are shown in Fig. 1 with a set of typical parameters, which will be demonstrated later.

The transport coefficient \( D \) and \( \chi \) are related by the formula \( D = 0.6\chi \) which is obtained in previous simulations of non-stationary particle transport, and supported by the ratio of the experimental particle and energy confinement times \[15\]. The thermal diffusivity \( \chi \) can be described as

\[
\chi = \chi_0 g(r), \tag{5}
\]

where \( g(r) \) is a function of \( r \), as is shown in Fig. 2. \( g(r) \) describes a fixed radial profile with an edge transport barrier (ETB) and the constant transport coefficient in the SOL. Note that this profile corresponds to a pedestal width of 0.05\( a \), which is reasonable according to Ref. \[19\]; \( \chi_0 \) is a constant.

In the simulations for studying the dependence of fusion performance on fuelling depth, the value of \( \chi_0 \) in every time step is obtained by iterating the transport equations [Eqs. (1)–(4)] and making sure that \( \chi_0 \) be such a value that makes the change of total plasma energy \( W \) meet the following formula in every time step:

\[
\frac{\partial W}{\partial t} = P - \frac{W}{\tau_E}, \tag{6}
\]

where \( P = \int (S_\alpha + S_{aux} - S_{rad})d^3x \) is the total heating power; \( W = \int 3nT d^3x \); \( \tau_E \) is the energy confinement time. According to the ITERH-98P(\( y,2 \)) scaling law \[4,5\],

\[
\tau_E = 0.0562H_{98p(\gamma,2)}^{0.93} B_T^{0.15} a_r^{0.41} P^{-0.69} R^{1.97}
\]
where \( H_{B98(r, 2)} \) is the confinement improvement factor; \( \rho_p \) is the plasma current; \( B_T \) is the toroidal magnetic strength; \( \bar{\pi}_e \) is the line averaged electron density; \( R \) is the plasma major radius; \( M \) is the average ion mass number; \( \kappa_a \) is the effective elongation; \( \epsilon \) is the inverse aspect ratio. The variables in Eq. (7) are treated in the same way as Ref. [7], which used a set of typical parameters of the ITER inductive scenario.

For the study of the dependence of particle confinement on fuelling depth, \( \chi_0 \) is fixed to a reference value.

The particle pinch velocity is derived from the turbulence equipartition (TEP) theory [20]:

\[
u_p = -D \frac{\frac{1}{2} + \frac{1}{3} \frac{r}{q} \frac{dn}{dr} - \frac{3}{r} \frac{r}{q} \frac{dn}{dr} dr}{R - \frac{r}{2} - \frac{3}{q} \frac{r}{5} \frac{dn}{dr} dr},
\]

where \( q \) denotes the safety factor. Note that the pinch velocity is set to be zero in the SOL region \((r > 1)\) since in this region there is no clear topological magnetic structure which is the basic of TEP theory. According to the inductive scenario of ITER [4], the safety factor is set to be a fixed monotonic form. The profile of the safety factor used in this paper and the corresponding pinch velocity steady profile with typical inductive scenario parameters are shown in Fig. 5(d).

\[n/\tau_{||} \text{ and } 3nT/\tau_{||} \text{ in Eqs. (1)-(2) represent the parallel loss in the SOL induced by the open magnetic force line. Assuming the particle source is negligible and the transport coefficient is a constant in the SOL, the particle balance in the SOL can be described as } [11]:

\[
\nabla \cdot \Gamma + \frac{n}{\tau_{||}} = 0,
\]

\[
\Gamma = -D_{\perp} \nabla n,
\]

where \( \Gamma \) is the diffusion induced particle flux and \( D_{\perp} \) is the perpendicular transport coefficient in the SOL. We can obtain a general solution of \( n \) with the slab geometry approximation:

\[
\begin{align*}
n &= n_0 \exp\left(-\frac{r - 1}{\lambda_n}\right), \\
\lambda_n &= \frac{\sqrt{D_{\perp} \tau_{||}}}{a},
\end{align*}
\]

where \( n_0 \) is an undetermined coefficient; \( \lambda_n \) is the decay length. Following Eq. (12) and the fact that there is no parallel loss term in the confined plasma region, \( \tau_{||} \) is given by:

\[
\tau_{||} = \left\{ \begin{array}{ll}
\infty, & \text{for } r \leq 1.0, \\
\frac{\lambda_n^2 r^2}{a}, & \text{for } r > 1.0.
\end{array} \right.
\]

\( \lambda_n \) is set to be 0.0065 (0.013 m) based on the empirical estimate in Ref. [9].

The boundary conditions of the simulation are set as:

\[
T(r_{wall}) = 0, \quad \left. \frac{dT}{dr} \right|_{r=0} = 0,
\]

where \( r_{wall} \) represents the radial position of the device wall, it marks the real endpoint of the computational domain. In this paper \( r_{wall} \) is set to be 1.1.

The initial temperature \( T_0(r) \) and density \( n_0(r) \) in the simulations are shown in Fig. 3.

![Fig.3 Radial profiles of the initial temperature and density](image)

![Fig.4 Radial profiles of the particle source](image)

The particle source distribution is assumed to have a form as:

\[
S_p = S_0 \exp\left(-\frac{(r-r_f)^2}{\Delta r^2}\right),
\]

where \( S_0 \) is the height; \( r_f \) is defined as the fuelling depth; \( \Delta r \) is the characteristic width. As is shown in Fig. 4, three kinds of particle source distribution with different \( r_f \) are used in this work to investigate the effects of fuelling depth on fusion performance and particle confinement. In the three kinds of particle source distribution, we keep \( \Delta r \) unchanged, and \( S_0 \) is adjusted to keep the fuelled particle number unchanged. It should be stressed that in this work the ETB, which can also stop most of the particles fuelled by the present fuelling devices from penetrating to the core region, is located at \( r = 0.95 \). Therefore the particle sources with \( r_f = 0.85 \) and \( r_f = 0.90 \) are still uncertain to achieve in ITER. To overcome the difficulty of fuelling penetration, the possible methods are raising the proportion of PI (according to Ref. [9], with the pellet velocities of up to 4 kms\(^{-1}\), its penetration up
to 20%–30% of the minor plasma radius in ITER) fuelled particles in total fuelling particles and applying a new deep fuelling method such as the compact toroid injection [9].

3 Results and discussion

With different fuelling and transport conditions, simulations are carried out in various cases. In all of the cases the system evolved to a steady state after a certain period of time.

3.1 Reference cases

To set a baseline for the comparisons in this work, we firstly modeled a case with the same fusion performance as the planned ITER inductive scenario. A moderate fuelling depth \( r_f = 0.9 \) is chosen and \( S_0 \) in Eq. (16) is varied to obtain a suitable volume averaged density. Under these conditions, a steady state with \( Q = 8.80 \) has been obtained along with the particle confinement time \( \tau_p = 2.37 \, \text{s} \). This reference case is referred to as case I in this paper. The steady state profiles in this case are shown in Fig. 5. Note that in this work \( \tau_p \) is calculated by the formula: \( \tau_p = N/S_{\text{tot}} \), where \( N = \int n \, d^3x \) is the total particle number; \( S_{\text{tot}} = \int S_p \, d^3x \) is the total particle source.

3.2 Effect of fuelling depth on fusion performance

To investigate the effect of fuelling depth on fusion performance, simulations with different fuelling conditions have been carried out. As is shown in Fig. 4, a deeper particle source \( (r_f = 0.85) \) and a shallower particle source \( (r_f = 0.95) \) are applied to the simulations which are referred to as case II and case III, respectively. Note that by keeping the total particle source \( S_{\text{tot}} \) unchanged [adjust \( S_0 \) in Eq. (16)], the only difference of the two cases with the reference case is the fuelling depth \( r_f \). Since the focus of this work is on the effect of fuelling depth, the transport coefficients in the three cases above have the same shape, which is shown in Fig. 2.

The steady state profiles and corresponding parameters are shown in Fig. 5 and Table 1, respectively.

| Label in Fuelling Average density \( Q \) \( \tau_p \) |
|---|---|---|---|
| I \( r_f = 0.9 \) | \( 1.01 \times 10^{20} \, \text{m}^{-3} \) | 8.80 | 2.37 s |
| II \( r_f = 0.85 \) | \( 1.15 \times 10^{20} \, \text{m}^{-3} \) | 11.59 | 2.69 s |
| III \( r_f = 0.95 \) | \( 0.79 \times 10^{20} \, \text{m}^{-3} \) | 5.57 | 1.84 s |

We can see that with 0.05a deeper fuelling depth, the fusion performance of case II is 31.8% higher than that of case I; the fusion performance of case I is 58.0% higher than that of case III. With 0.1a deeper fuelling depth, the fusion performance of case II is 108.3% higher than that of case III. We conclude that fuelling depth significantly affects the fusion performance of a fusion reactor, with the same total particle source injected into the burning plasma, deep fuelling depth will be beneficial for high fusion gain factor.

**Fig. 5** Steady-state profiles for the scaling law dependent cases. (a) Density, (b) Temperature, (c) Pressure, (d) Safety factor and pinch velocity
3.3 Effect of fuelling depth on particle confinement

Note that the transport coefficients in the simulations above depend on the global plasma parameters; the particle confinement time in these cases is possibly affected by other elements in addition to the fuelling depth. To investigate the direct effect of fuelling depth on particle confinement, we simulate the density evolution with fixed transport coefficient of the steady state in case I is chosen to use for the following simulations. The resulted steady state density profiles and parameters are shown in Fig. 6 and Table 2, respectively.

![Density Profiles](image_url)

**Fig. 6** Steady-state density profiles for the fixed transport coefficient cases

| Table 2. Steady-state parameters of the fixed transport coefficient cases |
|-----------------------------|----------------|----------------|----------------|
| **Label in Fig. 6** | **Fuelling depth** | **Average density** | **τp** |
| I | 0.9a | 1.01 \times 10^{20} \text{ m}^{-3} | 2.37 s |
| II' | 0.85a | 1.20 \times 10^{20} \text{ m}^{-3} | 2.81 s |
| III' | 0.95a | 0.74 \times 10^{20} \text{ m}^{-3} | 1.74 s |

We can see that although the particle confinement time in these cases is slightly different from that of the scaling law dependent cases, it shows the same tendency that deeper fuelling depth leads to better particle confinement. With 0.05a deeper fuelling depth, the particle confinement time of case II' is 18.6% longer than that of case I; the particle confinement time of case I is 36.2% longer than that of case III'. With 0.1a deeper fuelling depth, the particle confinement time of case II' is 61.4% longer than that of case III'.

3.4 Effect of fuelling depth with the same volume averaged density

In section 3.2 and section 3.3, the effects of fuelling depth on fusion performance and particle confinement were investigated by changing \( r_f \) with the same \( S_{\text{tot}} \); this configuration consequently leads to steady states with different volume averaged densities in comparisons. However, the volume averaged density (approximately 1.0 \times 10^{20} \text{ m}^{-3}) in the reference case is a design point of the ITER project, thus study of the effect of fuelling depth with the reference averaged density is necessary. By adjusting \( S_0 \) in Eq. (16), steady states with \( r_f = 0.85 \) and \( r_f = 0.95 \) are obtained along with the same averaged density, they are referred to as case I' and case I", respectively. The steady state profiles and corresponding parameters of them and case I are shown in Fig. 7 and Table 3. Note that the transport coefficients in these cases also have the same shape which is shown in Fig. 2.

| Table 3. Steady-state parameters in the scaling law dependent case |
|-----------------------------|----------------|----------------|----------------|
| **Label in Fig. 7** | **Fuelling depth** | **Average density** | **Q** | **τp** |
| I | 0.9a | 1.01 \times 10^{20} \text{ m}^{-3} | 8.80 | 2.37 s |
| I' | 0.85a | 1.01 \times 10^{20} \text{ m}^{-3} | 9.36 | 2.84 s |
| I" | 0.95a | 1.01 \times 10^{20} \text{ m}^{-3} | 8.26 | 1.72 s |

It can be seen that the different fuelling depths also lead to different fusion performances and particle confinements with the same volume averaged density. For the particle confinement, the disparities among the different fuelling cases are similar with those of the cases in section 3.3. For the fusion performance, with 0.05a deeper fuelling depth, the fusion performance of case I' is 6.4% higher than that of case I; the fusion performance of case I is 6.5% higher than that of case I'. With 0.1a deeper fuelling depth, the fusion performance of case I' is 13.2% higher than that of case I'. In the cases with certain referenced volume averaged density, the benefit on fusion performance of the deeper fuelling depth is not as significant as those in the cases with the fixed total particle source. This is a reasonable phenomenon since for the steady states in section 3.2 the different volume averaged densities lead to different total fusion fuel, which will certainly result in different fusion powers. In the cases with the same volume averaged density, by contrast, the fuelling depth affects the fusion performance by its influence on density profile shape: the deeper fuelling depth leads to more peaked density profile. From Figs. 7(a) and (d) we can see that there is a tendency that the density profile in the case with deeper fuelling depth is more peaked, although the pinch velocities in the three cases are almost the same. With the same volume averaged density, the more peaked density profile can lead to higher temperature in the core region, this tendency is shown in Fig. 7(b).
3.5 Discussion

From Figs. 5(c) and 7(c) we can see that the fusion gain factor is clearly dependent on the pressure profile; in section 3.2 the pressure profiles are significantly different in the three cases while the temperature profiles are almost the same. Thus we can attribute the difference of $Q$ to the difference of density profiles. From the results in section 3.3 and section 3.4, it can be seen that the density profile has a notable dependence on the fuelling depth, thus we can see the clear relationship among these elements: the deeper fuelling leads to higher particle density (with the same total particle source) or more peaked particle density profile (with the certain volume averaged density); the higher (or more peaked) density leads to higher pressure; the higher pressure leads to higher $Q$. This relationship can also be seen in Ref. [7] where the deeper fuelling depth brings in higher $Q$, although it does not change the dependence of $Q$ on the pinch effect.

The relationship between density and fuelling depth is easy to understand, since the particles fuelled in a deeper location will experience a longer time to be transported out, thus they have a longer confinement time under the same transport condition, which leads to the density rising. Note that in Ref. [12], based on the experimental data, there has been proposed an empirical formula for particle confinement time:

$$\tau_p \approx \frac{a \lambda}{D},$$

(17)

where $\lambda$ is the penetration depth of fuelled particles. Here we define $\lambda = r_{\text{wall}} - r_f$ and assume that $D$ in Eq. (17) is a constant for the different particle source distributions. From Fig. 2 it can be seen that the assumption on $D$ is reasonable, since the transport coefficient does not change significantly from $r = 0.85$ to $r = 0.95$. Hence, from Eq. (17) we can obtain a proportional relationship for the particle confinement time with different $r_f$:

$$\frac{\tau_p}{r_f=0.85} : \frac{\tau_p}{r_f=0.90} : \frac{\tau_p}{r_f=0.95} = 5 : 4 : 3. \quad (18)$$

According to the data in Tables 1–3, in the scaling law dependent cases this proportion is $4.39 : 3.86 : 3$; in the fixed transport coefficient cases it is $4.84 : 4.09 : 3$; in the cases with the same averaged density, it is $4.95 : 4.13 : 3$. It can be seen that the proportional relationships of particle confinement times calculated in our simulations show a good qualitative agreement with the experimental result in Ref. [12]. It is reasonable that the results in the latter two situations are more consistent with Eq. (18), since the conditions of them are more consistent with our assumption on $D$. In the fixed transport coefficient cases the simulations are carried out with the same $D(r)$; in the cases with the same averaged density, the global parameters in every simulation are very close to each other, therefore, $\chi_0$ in every simulation are very close to each other, which makes $D(r)$ close to each other in every simulation.

4 Summary

The effect of fuelling depth on fusion performance and particle confinement of a fusion reactor has been
studied by modeling the temperature and density transport. The ITERH-98P(y,2) scaling law has been used to evaluate the transport coefficients and the parallel loss in the SOL has been taken into account. From the numerical results of the transport modeling, one can draw the following conclusions:

a. The fuelling depth can significantly affect the fusion performance. With the same total particle source, changing the fuelling depth by 5% of the minor radius can make the fusion performance change by 31.8%–58.0%; changing the fuelling depth by 10% of the minor radius can make the fusion performance change by 108.3%. With the same volume averaged density, changing the fuelling depth by 5% of the minor radius can make the fusion performance change by 6%; changing the fuelling depth by 10% of the minor radius can make the fusion performance change by 13.2%.

b. The particle confinement significantly depends on the fuelling depth. With the fixed transport coefficients, changing the fuelling depth by 5% of the minor radius can make the particle confinement time change by 18.6%–36.2%; changing the fuelling depth by 10% of the minor radius can make the particle confinement time change by 61.4%.

c. The deeper fuelled particles will have longer confinement time under the same transport conditions. Therefore, deeper fuelling can significantly increase the particle density in the core region plasma and consequently increase the plasma pressure, leading to enhanced fusion performance.

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