Hydrodynamics of Exploding Foil X-Ray Lasers with Time-Dependent Ionization Effect

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Abstract A simple modified model is presented based on R. A. London’s self-similarity model on time-independent ionization hydrodynamics of exploding foil X-ray lasers. In our model, the time-dependent ionization effect is under consideration and the average ion charge depends on the temperature. Then we obtain the new scaling laws for temperature, scale length and electron density, which have better agreement with experimental results.

Keywords: ionization, self-similarity model, foil, X-ray lasers

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1 Introduction

In this work, we study the hydrodynamics of exploding foil X-ray lasers. It has been widely studied due to its great importance in the field of the interaction between lasers and plasma. In 1984, soft X-ray lasers were first demonstrated in the Lawrence Livermore National Laboratory [1,2], where the foil target was successfully used in experiments. Due to the large driven power needed by the foil target, people began to look for new methods to reduce the driven power in producing X-ray lasers (XRL). In 1987, the Naval Research Laboratory first used a slab target to reduce the driven power needed to $10^{12}$ W/cm² [3], which made a breakthrough to the foil target of LLNL [4]. Then, with the wide use of the pre-pulse technique [5–7], the driven power needed for the production of XRL could be further decreased. However, there are also some problems with slab targets experiments. On one hand, the deflection angle of slab targets changes with the target parameters while foil targets have no deflection angles [8]. On the other hand, the maximum output energy produced by slab targets is lower than that of foil targets. In the existing literatures [9,10], the output energy of the foil target can reach 7 mJ while the maximum output energy of the slab target is about 4 mJ, which is nearly half of the foil targets’ output. Consequently, the foil target is still a good choice when the driven power is ignored, the experiments are more easily performed, and the required output of energy is huge [11]. To study the hydrodynamics of exploding foil X-ray lasers, R. A. London and Rosen made a self-similarity model on the isothermal expansion process of the foil target in the corona region [12]. In this model, the experiment process was divided into three stages. The first stage is between the lasers reach at the target and the target gets transparent to the lasers, the last stage is the process of the target cooling down, and the second one is between them. In the three stages, the relations between the temperature, the scale length, the electron density and time are investigated, respectively. They also assumed that the average ion charge was constant in both time and space, which ignored the ionization effect varying with time in the experiments. This assumption is not in good agreement with the experiment due to the fact that the average ion charge varies with the temperature. Although London’s model is roughly right, the results are not exactly precise. Here we give a modified model where the relationship between the average ion charge $Z$ and the temperature $T$ is under consideration. Our model could give more precise and practical results.

2 Details of the self-similarity equations

In our model, the process of XRL’s production by laser driven irradiation foil targets is still divided into three stages, and the ionization effect is under consideration. The first stage is that the laser works on the target and produces plasma until it gets transparent. In the meantime, the ionization effect absorbs energy and affects the temperature. The second one is that the laser continues heating the
plasma to keep it transparent and isothermal. The last one is that the laser stops heating and the plasma begins to expand adiabatically, during which the temperature and the ionization degree reduce.

In the whole process, the average ion charge is still assumed to be constant in space, which is in agreement with London’s assumption. But in the time scale it is no longer a constant. When the XRL is produced, the electron density in plasma is close to the electron density in the corona area, the relationship of the average ion charge and the temperature can be described by the formula of the ionization degree in the corona area. We introduce the ionization degree formula in the corona area \[13; \]

\[
Z = \frac{20}{3} [AT]^{1/3}, \tag{1}
\]

where \(Z\) is the average ion charge, \(A\) is the atomic mass and \(T\) is the temperature with the unit keV. Taking the normalized values for scaled variables into Eq. (1), then we can get

\[
Z = 225 = \frac{20 \times 80}{3} [AT]^{1/3}, \tag{2}
\]

where \(Z\) is the normalized variable of \(Z\). In this paper, the underlined variables are the corresponding normalized variables and the normalizing values are shown in Table 1. The equations of state and heat capacity (EOS) are calculated assuming an ideal gas and we have

\[
p = ZT \rho/M_i, \tag{3}
\]

\[
C_v = \frac{3}{2} Z/M_i, \tag{4}
\]

where \(M_i, \rho, p\) and \(C_v\) are the ion mass, density, pressure and specific heat per unit mass, respectively. For the short pulse (fs or ps), the ionization process is too short to go on due to the short reaction time, so we take the average ion charge \(Z\) as a constant in this situation. For the long pulse (ns) we study, the ionization process cannot be ignored so \(Z\) cannot be taken as a constant any more. In this case, substitute Eq. (1) into Eqs. (3) and (4) then we get

\[
p = \frac{20}{3} A^{1/3} T^{4/3} \rho/M_i, \tag{5}
\]

\[
C_v = \frac{10 A^{1/3} T^{1/3}}{M_i}. \tag{6}
\]

Then we give a modification to the self-similarity equations in London’s model and we can get the time-dependent ionization self-similarity equations as follows

\[
L \frac{d^2 L}{dt^2} = \frac{20}{3} \frac{A^{1/3} T^{4/3}}{M_i}, \tag{7}
\]

\[
\frac{30 A^{1/3} T^{4/3}}{4} \frac{dL}{dt} = H - \frac{20}{3} \frac{A^{1/3} T^{4/3} dL}{M_i \frac{dL}{dt}}, \tag{8}
\]

where \(L\) is the time-dependent scale factor and \(t\) time.

### Table 1. Normalized values for scaled variables

<table>
<thead>
<tr>
<th>Physical variable</th>
<th>Symbol</th>
<th>Normalized value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>(t)</td>
<td>1 ns</td>
</tr>
<tr>
<td>Laser intensity</td>
<td>(I)</td>
<td>(10^{14}) W·cm(^{-2})</td>
</tr>
<tr>
<td>Laser wavelength</td>
<td>(\lambda)</td>
<td>1.06 (\mu)m</td>
</tr>
<tr>
<td>Foil column density</td>
<td>(m)</td>
<td>(10^{-2}) g·cm(^{-2})</td>
</tr>
<tr>
<td>Atomic mass</td>
<td>(A)</td>
<td>80</td>
</tr>
<tr>
<td>Coulomb logarithm</td>
<td>(\Lambda)</td>
<td>5</td>
</tr>
<tr>
<td>Laser focal spot width</td>
<td>(W)</td>
<td>200 (\mu)m</td>
</tr>
</tbody>
</table>

## 3 Analytical solutions of self-similarity equations

### 3.1 Early time: \(t < t_{\text{trans}}\)

In the first stage, when the laser irradiates the surface of the foil target, the optical depth is \(\tau_{ib} \gg 1\) and the net heating rate per unit mass is \(H = I/m\), where \(I\) is the total laser flux and \(m\) is the total column density of the foil. From Eqs. (7) and (8) we have the following power law functions of scale length \(L\), temperature \(T\) and electron density \(n_e\) that satisfy the new self-similarity equations:

\[
L = 7.127 \times 10^{-2} (\text{cm})^{1/2} m^{-1/2} \lambda^{3/4}, \tag{9}
\]

\[
T = 6.07 (\text{keV}) m^{-3/4} \Lambda^{1/4} L^{-5/4}, \tag{10}
\]

\[
n_e = 2.21 \times 10^{20} (\text{cm}^{-3}) m^{-1/4} \Lambda^{-1/4} L^{-5/4}. \tag{11}
\]

### 3.2 Mid-time: \(t_{\text{trans}} \leq t \leq t_L\)

The expression of the optical depth is

\[
\tau_{ib} = D 3.846 \times 10^{-2} m^{2/3} \Lambda^{2/3} \lambda^{-2} T^{-3/2} L^{-1}, \tag{12}
\]

where the critical density is \(n_c = 1.1 \times 10^{21} (\text{cm}^{-3}) \lambda^{-2}\), \(\lambda\) the laser wavelength, \(\Lambda\) the coulomb logarithm, and \(D = (1 - n_e/n_c)^{-1/2}\).

When the foil target becomes transparent to the laser light at the transparent time \(t_{\text{trans}}\), we have the optical depth \(\tau_{ib} = 1\). Then we can obtain the transparent time:

\[
t_{\text{trans}} = D^{8/15} 0.55566 L^{-7/15} m^{23/15} \Lambda^{-2/3} \lambda^{8/15} \Lambda^{16/15}. \tag{13}
\]

The temperature at the transparent time is

\[
T_{\text{trans}} = D^{2/3} 3.91 (\text{keV}) L^{2/5} m^{2/5} \Lambda^{2/5} \lambda^{4/5}. \tag{14}
\]

In the stage \(t_{\text{trans}} \leq t \leq t_L\), the laser heating gets balanced by the plasma expansion cooling, so the temperature is nearly kept constant. Therefore, we still assume the temperature in this stage is varying quite slowly, i.e., \(\frac{dT}{dt} \ll 1\). From Eqs. (7) and (8) we can get the following solutions:

\[
T = 1.3554 T_{\text{trans}} \ln \frac{L}{L_{\text{trans}}} + 4.5745 \left(1 - \frac{1}{5}\right). \tag{15}
\]

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Here we have the slope of our temperature again become lower. The ionization degree still plays a role and this makes the target to get transparent. In the third stage, the result in the second stage and it takes a longer time for the temperature in our model becomes lower than London’s model. Consequently, the temperature in our model is dependent on temperature in our model, while it is constant in London’s model. Then we can get

$$n_e = \frac{T_{\text{trans}}}{L_{\text{trans}}} \ln \left( \frac{L}{L_{\text{trans}}} \right) + \left( \frac{n_0}{n_{\text{trans}} L_{\text{trans}}} \right)^{30/13} - 13/30,$$

where $n_{\text{trans}}$, $L_{\text{trans}}$, and $T_{\text{trans}}$ are the electron temperature, scale length, and electron density at $t_{\text{trans}}$, respectively. Here we have

$$n_0 = D^{-2/15}L_{\text{trans}}^{1/15} \times 10^{20} \text{cm}^{-3}.$$

### 3.3 Late time: $t > t_L$

When the laser heating stops at $t_L$, the foil expands adiabatically. Then we have

$$L = L_L(t/t_L),$$

$$T = T_L(t/t_L)^{-2/3},$$

$$n_e = n_L(t/t_L)^{-11/9},$$

where $t_L$, $L_L$, and $n_L$ are electron temperature, scale length and electron density at the laser stopping time $t_L$ when the laser is turned off. The normalizing values for scaled variables are listed in Table 1.

### 4 Results and discussions

In the following part, we investigate the temporal history for temperature, scale length and ion charge and the position profile for electron density.

From Fig. 1 we can see that the trends of our lines are similar to London’s. The specific differences between our lines and London’s are as follows. In the first stage, the slope of our line is smaller than London’s. Physically, the temperature goes up with time slower because ionization absorbs energy; the ionization degree is dependent on temperature in our model, while it is constant in London’s model. Consequently, the temperature in our model becomes lower than London’s result in the second stage and it takes a longer time for the target to get transparent. In the third stage, the ionization degree still plays a role and this makes the slope of our temperature again become lower.

### Fig. 1

The blue solid line is our new temperature for the three stages. The black solid line is London’s numerical solution. The others are London’s analytical solutions.

Fig. 2 shows our scale length. In the first stage our scale length increases more slowly than London’s. In the last two stages, our results are closer to numerical solutions than London’s analytical solutions. London’s model assumes the ionization degree to be constant in the first stage while in our model the time-dependent ionization takes place during this stage and some of the energy is spent on it, which explains why the scale length increases slower. In the third stage our scale length is larger than London’s, which represents the smaller density gradient in plasma, and this is helpful to the amplification and propagation of X-ray lasers.

### Fig. 2

The blue dash-dot line is our scale length for all three stages. The black solid line is London’s numerical solution. The others are London’s analytical solutions for the three stages.

Fig. 3 illustrates the ionization degree in the whole process. London assumed the ion charge $Z$ to be a constant value in the whole process, while our model shows the ionization varies with time. Here we calculated the average ion charge value at $t_L$ with the formula $Z = (20/3) \times (AT)^{1/3}$ and it is about 29.95, which is nearly 20% higher than London’s numerical...
simulation result. This may be because although this formula suits most situations, it is applied better for heavy elements than light ones \cite{13}. For Ni-like XRL, it suits quite well \cite{14}; for Ne-like Se, which is the light element, it may be not so accurate. Hence, here we choose the formula $Z = 25 \times T^{1/2}$ to calculate the average ion charge, for this formula is predicted by London and it only suits the Ne-like Se XRL well. Using this formula we can get $Z = 26.786$ at the time $t_t$. From Fig. 3 it can be seen that the line drawn from $Z = 25 \times T^{1/2}$ agrees well with the simulation of LASNEX in the first and second stages and it varies with time. Compared with London’s assumption $Z = 24$, which is time-independent and always kept constant in all the stages, our model with the time-dependent ionization suits the practical situations better. In our model, the ionization degree changes with temperature. When laser stops heating in the third stage, the plasma goes on an adiabatic expansion, which causes the temperature to reduce, and then follows the ionization degree. Other research shows the same conclusion that the ionization degree rises nonlinearly as the temperature increases \cite{15}. The ionization degree is relatively high when the temperature is high and when the temperature becomes lower the ionization degree also becomes lower.

![Fig. 3](image_url)

**Fig. 3** The blue dash-dot line is our result of the formula $Z = 25 \times T^{1/2}$. The green dash-dash line is the ion charge drawn from London’s assumption $Z = 24$ and the black solid line is the simulation result of LASNEX

Usually, we focus on the second stage which is of most interest for X-ray lasers and other applications. Fig. 4 shows the comparison of electron density among our model, London’s model and the LASNEX simulation at $t = 0.45$ ns \cite{12}. We can see that the changing trends of all lines are similar but our result agrees better with LASNEX’s than London’s. As we all know, LASNEX considers most of the conditions in experiments and it basically represents practical situations. So our results are proved to be closer to the real situations than London’s and this is because the time-dependent ionization effect plays an important role in our model.

![Fig. 4](image_url)

**Fig. 4** The blue dash-dot line is our electron density at $t = 0.45$ ns. The others are the result of LASNEX and London’s model, respectively

Figs. 5 and 6 are the electron density at 0.2 ns and 0.35 ns in stage two, respectively. From the two figures, it can be seen that our electron density is flatter than London’s at the top and wider at other positions. This is because London’s model considers no ionization effect existing in producing XRL, which therefore causes larger error in calculation. Physically, the flatter the electron density gradient is, the more favorable it is to the propagation and amplification of X-ray lasers. Experiments show that the gain length can reach up to 3.8 cm \cite{9}. If the electron density gradient in the gain area is steep, the XRL will soon deflect out of the gain area and the gain length will not be so long. Only when the gradient is flat, can the XRL well propagate in the gain area and the gain length be long enough. So in this way, our model is closer to experiment results than London’s.

![Fig. 5](image_url)

**Fig. 5** The dash-dot line shows our electron density at $t = 0.2$ ns and the solid line is London’s result
Fig. 6 The dash-dot line shows our electron density at $t = 0.35$ ns and the solid line is London’s result.

5 Conclusions

In conclusion, we have modified average ion charge in the London self-similarity model and considered the time-dependent ionization effect in practical situations to make the characteristic quantities more suitable for most experiments. Due to the consideration of the average ion charge formula, our model can be suitable for both heavy elements and light elements \([13]\); it can also give a nice guidance to the practical experiment designs with the output of high energy.

References


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