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To cite this article: Wenjia WANG et al 2019 Plasma Sci. Technol. 21 015101

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The residual zonal flow in tokamak plasmas with a poloidal electric field

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Received 10 July 2018, revised 24 August 2018
Accepted for publication 29 August 2018
Published 9 November 2018

Abstract
In a tokamak plasma with auxiliary heating by cyclotron waves, a poloidal electric field will be produced, and as a consequence influence the residual zonal flow (RZF) level. The poloidal electric field can also be induced through biasing electrodes at the edge region of tokamaks. Numerical evaluation for a large aspect ratio circular cross section tokamak for the electron cyclotron wave heating indicates that the RZF level decreases significantly when the poloidal electric field increases. Qualitatively, the ion cyclotron wave heating is able to increase the RZF level. It is difficult to apply the calculation to the real cyclotron wave heating experiments since we need to know factors such as the plasma profiles, the exact power deposition and the cross section geometry, etc. It is possible to use the cyclotron wave heating to control the zonal flow and then to control the turbulence level in tokamak experiments.

Keywords: zonal flow, poloidal electric field, cyclotron wave

1. Introduction
In magnetic confinement devices, zonal flows (ZFs) are low frequency, long wave length perturbations symmetric both poloidally and toroidally. They are excited by the drift wave turbulence and can regulate the turbulence level on the other hand [1, 2]. Because ZFs play an important role in plasma turbulence evolution, it is very important to study its driving, damping and time evolution for determining the turbulence level of the tokamak discharge. Rosenbluth and Hinton (R–H) pointed out that in collisionless plasmas the ZF is not damped, i.e. Landau damping can be ignored because the parallel wave vector is zero [3], and the transit resonance is weak since the ZFs’ frequency is much smaller than the transit/bounce frequency of thermal ions. The long time evolution of the flow (much longer than the ion transit/bounce time) is determined by the neoclassic polarization. In the large aspect ratio circular cross section approximation it can be expressed as

$$\phi(t) = (1 + 1.6q^2/\sqrt{\varepsilon})^{-1}\phi(0),$$

(1)

where $\varepsilon$ is the inverse aspect ratio and $q$ is the safety factor. In later publications [4–6], this result is called the residual zonal flow (RZF). The RZFs have been investigated by many researchers from different aspects. Xiao et al [4, 5] and Zhou et al [6] considered the shaping effect on RZFs using different equilibrium models. It was revealed that the residual level increases with the plasma elongations. The evolution of ETG mode driven short wave length ZFs was studied by Kim et al [7] and it was found that the shearing of electron scale ZFs is not a robust mechanism to suppress the electron scale turbulence. Kagan and Catto [8] studied the RZFs in tokamak pedestal region and revealed that for extremely large electric fields the ratio between the residual flow and the initial flow is close to one. The effect of magnetic flutter on residual ZFs was investigated by Terry et al to account for the disruption of flows by stochastic magnetic fields [9], Sugama and Watanabe studied the dynamics of ZFs in helical systems [10]. The evolution of ZFs in a toroidally rotating plasma was numerically simulated by Casson et al [11], and an analytical expression of flow residual in a large aspect ratio toroidally rotating tokamak was given by Zhou [12, 13]. The RZF level decreases with the increasing rotating velocities. The ZFs’ dynamics in collisionless plasmas has also been studied in some works [5, 7, 14]. In the present work, we investigate the residual ZFs in a tokamak plasma with an equilibrium poloidal electric field.

In a tokamak plasma with auxiliary heating by ion cyclotron radio frequency waves (ICRH) or electron cyclotron radio frequency waves (ECRH), a poloidal equilibrium electric field can
be produced due to the piling up of particles at the lower field side [15]. This poloidal electric field was considered to be the cause of the density drop (increase) in ECRH (ICRH) experiments. In tokamak experiments, a poloidal equilibrium electric field can also be induced by application of biasing electrodes at the edge region. Marchenko explained the excitation of geodesic acoustic modes (GAMs) in ICRH by taking account of this poloidal polarization [16]. On DIII-D, electron cyclotron waves were injected and deposited on the surface of the minimum safety factor to suppress the reversed-shear Alfvénic eigenmodes (RSAEs) [17]. The poloidal electric field was adopted to explain the suppression of the RSAEs by Marchenko and Baschenko [18].

In ECRH experiments, an electric potential well forms on the lower field side in a tokamak because of the piling up of electrons while a potential hill forms in ICRH experiments. It is of great uncertainty to numerically evaluate the induced poloidal electric field related to the cyclone wave heating because a lot of parameters are involved (equation (2)). In this work, we assume that the poloidal electric field is prescribed and study the effect of this field on the RZF level.

In section 2, we give the physics model and the general formulation for RZF s. The numerical evaluation of RZF s for the tokamak with a large aspect ratio is shown in section 3. Summary is given in section 4.

2. The physics model and the general formulation for RZF s

In a tokamak plasma with cyclotron wave heating, the poloidal varying electric potential is given for a circular cross section tokamak by [15, 16, 18]

\[
e\Phi_p\frac{1}{T} = \frac{1}{\nu} e^{\Phi_0} \frac{1}{\nu} \epsilon \cos \theta,
\]

where \(e^{\Phi_0}\) is the quasilinear diffusion in velocity space induced by cyclotron waves, \(v\) is the collisional frequency, \(\epsilon\) is the inverse aspect ratio, \(\theta\) is the poloidal angle, \(\epsilon\) is the absolute value of the electron charge. \(\alpha\) is a numerical factor of order unity, it is negative for ECRH and positive for ICRH. In the present work, we study the collisionless evolution of the ZFs, which means that the collisional frequency is much smaller than the bounce/transit frequency of ions. To make the analytical treatment applicable, we take the weak rf limit, i.e. \(\nu \ll 1\). To numerically evaluate the poloidal electric field using equation (2) is a cumbersome work which needs the modeling of rf heating and involves the plasma profiles, the power depositions and the heating scenarios (basic harmonic or second harmonic absorption) etc [15]. So presently, we limit the scope of the study to the influence of a prescribed poloidal electric field. We study the case for ECRH and introduce \(e^{\Phi_0} = |\epsilon| e^{\Phi_0} \nu \sim \epsilon\). Then the electric potential is written as

\[
e\Phi_p\frac{1}{T} = -\frac{e^{\Phi_0}}{T} \cos \theta.
\]

The equilibrium ion distribution is in the form

\[
F_{10} = \frac{n_i}{\pi \nu v_{th}} \frac{1}{v_{th}} e^{-(n_i v_{th}/2 + e^{\Phi_0})/T_i}
\]

The unperturbed ion density is then given by

\[
n_0 = n_i e^{\Phi_0}/T_i = n_i e^{\Phi_0} \cos \theta/T_i.
\]

The plasma responds to an initial axisymmetric perturbation driven by turbulence in cyclotron time scale according to the classic polarization. In the present work, we neglect finite beta effect and adopt an electrostatic approximation. This approximation is valid for evaluation of the geodesic continuum or neoclassical polarization for symmetric perturbations in low beta limit, but not valid for Alfvénic eigenmodes [18, 19]. Although a few studies considered the electromagnetic effects for ZFs and GAMs in recent years [20–24], most relevant works still focused on the electrostatic perturbations. We consider a turbulence driven by ion temperature gradient and neglect the electron response. The ion density perturbation due to the classic polarization and the initial density source satisfy the quasi-neutrality condition, i.e. \(\rho_{pol}(0) + e_\Delta n(0) = 0\), where \(\rho_{pol}(0) = -e_\Delta \phi(0)/\epsilon\) the susceptibility from classic polarization in SI unit, \(e_\Delta n(0)\) the nonlinear initial ion density driven by turbulence. For a long time scale (longer than transit/bounce time) the neoclassical polarization due to the finite orbit width of ions in tokamaks should be included, and hence the long time potential is given by \(\phi(0) = (\phi_0 + \epsilon \phi_\Delta)\). One needs to solve the gyro-kinetic equation as an initial value problem to get the susceptibility \(\epsilon_{\text{neq}}\) from the neoclassical polarization.

The total drift velocity of ions satisfies \(v_\perp = -v_{\parallel} \times \nabla (v_\parallel/\omega_i)\) for a low beta plasma, with \(v_\parallel = (2/m_i)^{1/2}(E - e\Phi_p)^{1/2}, E = m_i v_{\parallel}^2/2 + e\Phi_p\) the total energy and \(\mu = m_i v_{\parallel}^2/2B\) the magnetic moment. For axisymmetric perturbations, the gyro-kinetic equation has the same form as that of R–H. One can solve the problem as done by R–H [3] or by adopting the Laplace transformation method [5, 14] to obtain the final result

\[
\phi(0) = \frac{1}{\epsilon_0} \omega_i (k_i \rho_i^2) (n_0) + \left( \int d^4 \psi (\mathbf{Q} - \mathbf{Q}) F_{\psi}, \right)
\]

where \(k_i = \nabla S(\psi)\) is the radial wave vector, \(Q = K v_{\parallel} / B\) with \(K = (m_{i} I / e) S(\psi)\) and the equilibrium magnetic field \(B = \nabla \phi \times \nabla \psi + I \nabla \psi, \rho_i = (T_i/m_i)^{1/2}/\omega_i\), the bracket denotes surface averaging, i.e. \((\langle .. \rangle) = \int \frac{d^4 \psi}{\epsilon_0} \), with the integral route completing a full poloidal circuit along the magnetic line, and \((\langle .. \rangle) = \int \frac{d^4 \psi}{\epsilon_0} \), defines the bounce/transit averaging, the integration is over a full poloidal orbit for trapped ions and over a poloidal circuit for passing ions.

3. Numerical evaluations

The form of equation (6) is the same as that of R–H [3, 14], but the numerical result is different. To make a quantitative
evaluation of equation (6), we consider a large aspect ratio tokamak with circular cross section. The magnetic field is \( B = B_0 / (1 + \varepsilon \cos \theta) \), and the major radius \( R = R_0 (1 + \varepsilon \cos \theta) \), with the inverse aspect ratio \( \varepsilon = r / R_0 \ll 1 \). The potential is assumed to have ordering \( e \Phi_0 / T_e \sim \varepsilon \), then it is straightforward to obtain

\[
\langle n_0 \rangle = \frac{2 \pi q \rho_{in} n_e}{B_0} (1 + \varepsilon \Phi / 2 + \ldots),
\]

(7a)

\[
\left\langle \int d^3 v \nabla^2 F_{0} \right\rangle = \frac{2 \pi q \rho_{in} n_e^2}{\varepsilon} \frac{\langle k_{\perp} \rho_e \rangle^2}{\varepsilon^3} \left( 1 + \varepsilon \Phi / 2 + \ldots \right),
\]

(7b)

where \( q = r B_0 / R_0 B_0 = r \) is the usual safety factor and the dimensionless potential \( \Phi = e \Phi_0 / T_e \). The expansion in small parameter \( \varepsilon \) is used to perform the \( \langle \cdots \rangle \) integration. To reach equation (7b), we have made use of the relation

\[
\int \frac{d \Omega}{|v|} \int d^3 v \nabla^2 F_0 \left( \frac{|v|}{B} \right) = \frac{\rho_{in}}{B_0} \int d^3 v \nabla^2 F_0 \left( \frac{|v|}{B} \right),
\]

(8)

which is ready to be confirmed noticing that \( d^3 v = \sum_{\sigma = \pm 1} 2 \pi \rho_0^2 d \mu d E \). The two integrations \( \langle \cdots \rangle \) and \( \langle \cdots \rangle \) are carried out under \( E \) and \( \mu \). Apparently, the term under the \( \langle \cdots \rangle \) operation on the left side of equation (8) is independent of position. It seems unnecessary to have this integration. However, to have this operation and reach the form as the right side integration, we can circumvent the more complex integration \( \int \frac{d \Omega}{|v|} \).

It is much involved to carry out the second part of the second integration in the denominator of equation (6). Using equation (8), we have

\[
\left\langle \int d^3 v \nabla^2 F_{0} \right\rangle = K^2 \int \frac{d \Omega}{|v|} \int d^3 v F_0 \left( \frac{|v|}{B} \right)
\]

(9)

Obviously trapped particles have no contribution to the integration. Transforming the integral to \( (E, \mu) \) space, equation (9) becomes

\[
\left\langle \int d^3 v \nabla^2 F_{0} \right\rangle = \frac{2 \pi K^2}{m_i} \left( \frac{d \Omega}{|v|} \right)^2 \int \frac{d \mu d E}{|v|}
\]

\[
\times \left[ \sum_{\sigma = \pm 1} \int d^3 v \left( \frac{d \Omega}{|v|} \right)^2 F_{0} d \mu d E \right].
\]

(10)

Since \( |v| = (E - \mu B - e \Phi_0) / \varepsilon = (E - \mu B + e \Phi_0 \cos \theta) \), we notice that a passing particle satisfies \( E > e \Phi_0 \). The passing particle region is shown in figure 1 in the \( (E, \mu) \) space. Introducing the pitch angle variable \( \lambda = \mu B_0 / E \) and defining \( \Lambda = 1 - \lambda \), we change the integration in velocity space to

\[
\int \frac{d \Omega}{|v|} d \mu d E = \int_{e \Phi_0}^{+\infty} d \tilde{E} \int_{e \Phi_0 / \varepsilon}^{1} d \lambda (\cdots) (E / B_0) d \Lambda.
\]

(11)

We make use of the Taylor expansion method [4, 6, 12] to carry out the integration in equation (10). Firstly, we have

\[
\int \frac{d \Omega}{|v|} = \frac{q R_0}{(2 / m_i)^{1/2} E^{1/2} N^{1/2}}
\]

\[
\times \int \frac{d \Omega}{|v|} \left[ 1 + \varepsilon \cos \theta / 2 \right] d \theta
\]

\[
\times \int \frac{d \Omega}{|v|} \left[ 1 + \varepsilon \cos \theta / 2 \right] d \theta
\]

\[
\times \int \frac{d \Omega}{|v|} \left[ 1 + \varepsilon \cos \theta / 2 \right] d \theta
\]

(12)

Let \( \tau = \Lambda^{-1}(\varepsilon + e \Phi_0 / E) \), obviously \( \tau \sim \varepsilon \) except for a very small region close to the passing-trapping boundary. Using the Taylor expansion formula \( (1 + x)^{-1/2} = 1 - x / 2 + 3x^2 / 8 - 5x^3 / 16 + 35x^4 / 128 + \ldots \), we can expand the denominator of equation (12) in powers of \( \tau \) to any desired order. Keeping up to \( O(\tau^4) \), we obtain

\[
\int \frac{d \Omega}{|v|} = \frac{q R_0}{(2 / m_i)^{1/2} E^{1/2} N^{1/2}} \left[ 1 + \frac{3}{16} \tau^2 + \frac{105}{1024} \tau^4 \right].
\]

(13)

Substituting equation (13) in equation (10), using equation (11), one reaches

\[
\left\langle \int d^3 v \nabla^2 F_0 \right\rangle = \frac{K^2 m_i T_i}{q R_0 m_i^{3/2} B_0} \left( \frac{2 \pi q R_0}{B_0} \right)^2
\]

\[
\times \int_{e \Phi_0}^{+\infty} d \Lambda \int_{e \Phi_0 / \varepsilon}^{1} d \lambda \left( \cdots \right) E^{3/2} N^{1/2} e^{-\tilde{E}},
\]

(14)

where a hat means normalization by ion temperature, i.e. \( \tilde{E} = E / T_i \) etc., and \( (\cdots) = 1 + 1 / 8 \varepsilon + \left( \frac{5}{512} \varepsilon^2 - \frac{3}{16} \varepsilon^2 \right)^2 \)

\[
+ \frac{3}{256} \varepsilon^3 \left( \frac{69}{1024} - \frac{27}{16384} \varepsilon^2 \right)^2 + \frac{1}{16} \varepsilon^2 + \ldots
\]

We have kept terms to the order \( O(\varepsilon^2) \) and \( O(\varepsilon^4) \). Carrying out the integration in equation (14), one obtains

\[
\left\langle \int d^3 v \nabla^2 F_0 \right\rangle = \frac{2 \pi q \rho_{in} \langle k_{\perp} \rho_e \rangle^2}{B_0}
\]

\[
\times \left[ 1 - 1.6 \varepsilon^{3/2} + \varepsilon^2 - \frac{3}{8} \varepsilon^5 / 2 - G(\Phi) \right]
\]

(15)
neoclassic polarization. When an electric potential well is
ion orbit lies on a magnetic surface and there is not the
normal component of the ion magnetic drift disappears, the
width of ions, and hence the result of the normal component
applying ECRH or biasing electrodes, the resulting radial
field increases, the residual level will decrease, as shown in
Figure 2. Physically it is easy to understand this phenomenon.

We have assumed the ordering \( \Phi \sim \varepsilon \ll 1 \), and kept terms up
to \( \varepsilon^{5/2} \). \( \text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2}dy \) is the well known error
function.

Inserting equations (7a), (7b) and (15) into equation (6),
one obtains

\[
\delta \Phi(t) = \frac{\delta \Phi(0)}{1 + \frac{q^2(1.603 - \varepsilon^{7/2} + 3\varepsilon)}{\varepsilon^{7/2}} q^5 \varepsilon + \frac{q^7 G(\Phi)}{(1 + \varepsilon \Phi / 2)^2} \varepsilon^7}. 
\]

(17)

The R–H result is recovered for \( \Phi = 0 \), as expected. As \( \Phi \)
increases, the residual level will decrease, as shown in
figure 2. Physically it is easy to understand this phenomenon.
The neoclassic polarization is the result of the finite orbit
width of ions, and hence the result of the normal component
of the magnetic drift of ions in a tokamak plasma. If the
normal component of the ion magnetic drift disappears, the
ion orbit lies on a magnetic surface and there is not the
neoclassic polarization. When an electric potential well is
formed on the lower field side of a tokamak no matter by
applying ECRH or biasing electrodes, the resulting radial
\( \mathbf{E} \times \mathbf{B} \) drift of ions is in the same direction of the radial
component of the ion magnetic curvature drift, which can be
readily checked. The neoclassic polarization is then enhanced.
As a result, the RZFs level will decrease with the increasing
electric field.

4. Conclusion and discussion

In this work we have calculated the RZFs for a tokamak
plasma with a poloidal electric field. A numerical evaluation
was given for the cases with a potential well on the lower field
side, which can be produced in ECRH experiments or by
biasing electrodes at the edge region. In these cases, the RZF
level decreases with increasing potentials. The physical
explanation has been presented at the end of section 3. For the
ICRH cases, it is expected that the ZFs’ residual level
increases with the increasing heating power due to the same
physical mechanism, i.e. the radial \( \mathbf{E} \times \mathbf{B} \) drift of ions is
opposite to the radial component of the ion magnetic curvature
drift, and hence the neoclassic polarization is reduced.

The present numerical results is only qualitatively
applicable to real cyclone wave experiments since in real
experiments many factors, such as plasma density and
temperature profiles, exact power deposition and plasma
geometric effects etc, should be taken into account. It is
difficult to quantitatively evaluate the ZFs’
residual for ICRH cases even for simplified geometry and given
gonal potentials because the equilibrium distribution of ions is not
represented by the simple form of equation (4), but depends
on the magnetic moment [15]. On the other hand, because the
electric potential well is formed on the high field side in ICRH
experiments, it is of difficulty to distinguish the trapping/
passing boundary. Hence, it becomes complicated to carry out
the integrals in equation (10). In real tokamak experiments, it is
possible to test these predicted effects through comparison of
plasma confinement, turbulence behavior or direct detection
of ZFs under different cyclotron wave heating scenarios.
Since it is widely believed that the L–H transition might be
triggered by the shearing of ZFs at the edge region, the pre-
sent work infers that the power threshold with ICRH heating
is smaller than that with ECRH heating under the same
experiment conditions.

In this work, we have employed the electrostatic
approximation for the axisymmetric perturbation under a
poloidally dependent background electrostatic field. Electromagnetic
effects were considered in a few recent works for
ZFs and GAMs. Catto et al revealed that a magnetic sideband
will appear for an initial electrostatic zonal perturbation, and
its amplitude is proportional to the plasma pressure ratio [22].
Although electromagnetic effects are interesting and some-
times bring about new phenomena, it is conventional and
convenient to adopt the electrostatic approximation in the
research of drift wave turbulence and ZFs for low beta plasma
limit.

Acknowledgments

This work is supported by National Natural Science Foun-
dation of China (No. 11675222).
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