Electron Acceleration by Microwave Radiation Inside a Rectangular Waveguide

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Abstract Electron dynamics in the fields associated with a transverse magnetic (TM) wave propagating inside a rectangular waveguide is analytically studied. The relativistic momentum and energy equations for an electron are solved, which was injected initially along the propagation direction of the microwave. Expressions of the acceleration gradient and deflection angle are obtained. In principle, it is shown that the electron can be accelerated in this condition and there is no deflection when the electron is injected from the centre of the waveguide front. However, it is found that the acceleration gradient and deflection angle depend strongly on the parameters of the microwave (intensity, frequency, etc.) and the dimensions of the waveguide.

Keywords: waveguide, electron acceleration, polarized microwave

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1 Introduction

The dynamics of a charged particle (an electron, for example) in the fields of an electromagnetic wave are fundamental in plasma physics, also of great interest due to its applications in the fields of laser-particle interactions, thermonuclear fusion, energetic particle physics, etc. Recently, considerable progress has been made with the problem of the acceleration of charged particles to high energy [1]. Different mechanics could be used to achieve high energy gain through direct acceleration in the fields of electromagnetic waves, plasma or laser wake-field acceleration and beat-wave acceleration. The direct acceleration scheme, as in microwave-plasma interaction experiments, has been realized by the Lorentz force.

A number of studies have been dedicated to electron acceleration by the interaction of laser and microwave fields [2-5]. PANTELL and SMITH [2] analyzed the interaction of a slow microwave signal and a laser light beam. They calculated the energy gradient of an electron as a function of fields of the electromagnetic waves. Electron acceleration under the collisionless condition by an oscillating electric field was investigated in an experimental system [6], where electrons were injected from the tip of an electron source along the magnetic field between two electrostatic potentials. MALIK [7] studied the electron trajectory in the resultant field of two fundamental transverse electric (TM_{10}) modes propagating along a rectangular waveguide and calculated the energy gain during the motion. JAWLA et al. [8] also studied the electron acceleration and evaluated the fields for fundamental \( H_{10} \) mode in a waveguide filled with plasma under the effect of an external magnetic field.

However, the properties of the waveguides, guiding the electromagnetic fields for a very long distance, have been extensively adopted for particle acceleration. Also, in view of the low cost and significant particle acceleration, it is better to use microwaves in place of ultrahigh lasers. So, in this study, the dynamics and the acceleration of an electron in the fields associated with TM_{11} modes, excited by a microwave propagating along a rectangular waveguide, are investigated. Expressions of the acceleration gradient and the energy gain are obtained, which are affected by the dimensions of the waveguide and the parameters of the microwave.

2 Dynamics of an electron in the fields of TM_{11}-modes

A hollow metallic tube, as the waveguide, with a rectangular cross section of width \( a \) along the \( x \)-axis and height \( b \) along the \( y \)-axis is assumed. The propagation of the microwave radiation of frequency \( \omega \) is made use of to excite TM_{mn} modes inside the waveguide. The field components associated with the mode can be obtained from Maxwell equations, with a time dependence as \( e^{i\omega t} \).

\[
\left( \nabla^2 + \frac{\omega^2}{c^2} \right) E = 0, \tag{1}
\]

\[
\left( \nabla^2 + \frac{\omega^2}{c^2} \right) H = 0. \tag{2}
\]

However, the TM_{mn} modes in a rectangular waveguide are characterized by \( H_z = 0 \) and the energy transmission in the guide is done by the \( z \)-component of the electric field. Consequently, Eqs. (1) and (2) are solved by considering the boundary conditions that the tangential component of the electric field \( E_\phi \) and the nor-
nal component of the magnetic field must vanish on the conducting surface. So, the field components of TM\(_{11}\) are

\[
E_x = \left( \frac{-i\pi \beta_g}{ak_c^2} \right) E_0 \cos(\pi x/a) \sin(\pi y/b)e^{-i\beta_g z},
\]

\[
E_y = \left( \frac{-i\pi \beta_g}{bk_c^2} \right) E_0 \sin(\pi x/a) \cos(\pi y/b)e^{-i\beta_g z},
\]

\[
E_z = E_0 \sin(\pi x/a) \sin(\pi y/b)e^{-i\beta_g z},
\]

\[
H_x = \left( \frac{i\pi \omega e}{b k_c^2} \right) E_0 \sin(\pi x/a) \cos(\pi y/b)e^{-i\beta_g z},
\]

\[
H_y = \left( \frac{-i\pi \omega e}{ak_c^2} \right) E_0 \cos(\pi x/a) \sin(\pi y/b)e^{-i\beta_g z}.
\]

In our case, \(\omega = 2\pi f\) and \(f > f_c\), where \(k_c\) and \(f_c\) are the wave number and frequency corresponding to the cutoff condition,

\[
f_c = \frac{1}{2\sqrt{\mu_0\varepsilon}} \left( \frac{1}{a^2} + \frac{1}{b^2} \right)^{1/2}, \quad k_c = \omega \sqrt{\mu_0\varepsilon}.
\]  

(4)

Also, \(\beta_g\) is the propagation constant given by \(\beta_g^2 = \frac{\omega^2 - \frac{\pi^2}{a^2} - \frac{\pi^2}{b^2}}{\epsilon}\) and \(E_0\) is the amplitude constant associated with the field mode.

It is seen from Eq. (4) that the cutoff frequency \(f_c\) depends on the width \(a\) and height \(b\) of the waveguide. This shows that the frequency of the mode, being able to propagate in a waveguide of dimensions, for example, \(a = 4\) cm and \(b = 2.5\) cm, should be higher than 7.15 GHz.

In previous theoretical studies the fundamental TE\(_{01}\) mode excited by a microwave either in an evacuated rectangular waveguide [6] or in a waveguide filled with plasma [4] was considered.

In this study the dynamics of an electron in the fields of another mode (TM\(_{11}\) mode) is studied. Let the electron be injected along the propagation direction of the mode inside the waveguide. At first, we calculate the resultant electric and magnetic fields in the plane perpendicular to the direction of propagation of the mode. The resultant electric field is obtained as

\[
E_{\perp} = -i \left( \frac{\pi \beta_g}{k_c^2} \right) E_0 e^{-i\beta_g z} \cos(\pi x/a) \cos(\pi y/b)
\]

\[
\times \left[ \tan^2(\pi y/b)/a^2 + \tan^2(\pi x/a)/b^2 \right]^{1/2}.
\]  

(5)

Also, the resultant magnetic field in the transverse plane is

\[
H_{\perp} = -i \left( \frac{\pi \omega e}{k_c^2} \right) E_0 e^{-i\beta_g z} \cos(\pi x/a) \cos(\pi y/b)
\]

\[
\times \left[ \tan^2(\pi y/b)/a^2 + \tan^2(\pi x/a)/b^2 \right]^{1/2}.
\]  

(6)


These resultant fields made angles \(\theta_1\) and \(\theta_2\), respectively, with the \(x\)-axis,

\[
\tan \theta_1 = \frac{a}{b} \tan \left( \frac{\pi x}{a} \right) \cotan \left( \frac{\pi y}{b} \right),
\]  

(7)

\[
\tan \theta_2 = \frac{b}{a} \cotan \left( \frac{\pi x}{a} \right) \tan \left( \frac{\pi y}{b} \right).
\]  

(8)

The angle \(\alpha\), at which the electron is deflected from the \(z\)-axis, can be evaluated from the following electron’s momentum equation,

\[
\frac{dP}{dt} = -e(\mathbf{E} + \mu_0 \mathbf{v} \times \mathbf{H}).
\]  

(9)

Taking into account the resultant field components in the equation of electron’s transverse motion, where \(dP/dz = -(e/v_z)(E_{\perp} + \mu_0(v \times H)_{\perp})\) can be integrated (by considering \(v_z\) to be slowly varying quantity), one obtains

\[
\tan \alpha = \frac{p_y}{p_x} = \left( \frac{-e\pi}{m_c k_c^2 v_z^2} \right) E_0 \cos(\frac{2\pi x}{\lambda_g})
\]

\[
\times \left[ \frac{\cos^2(\pi x/a) \sin^2(\pi y/b) + \sin^2(\pi x/a) \cos^2(\pi y/b)}{a^2} \right]^{1/2}
\]

\[
- \left( \frac{\omega e v_z \mu_0}{k_g} \right) \left[ \frac{\sin(\pi x/a) \cos(\pi y/b)}{b} + \frac{\cos(\pi x/a) \sin(\pi y/b)}{a} \right].
\]  

(10)

From this expression it can be seen that the deflection angle depends strongly on the injection point of the electron at the entrance of the waveguide in the \(x\)-\(y\) plane. In addition, it also depends on the initial energy of the injected electron (or velocity \(v_z\)), the frequency \(f\) and the intensity \(E_0\) of microwave.

Here, \(E_0\) is the maximum electric field of the mode, related to the intensity of the microwave radiation, and \(v_z\) is the velocity of the electron, related to its energy, along the \(z\)-axis. It can also be seen from Eq. (10) that the deflection angle \(\alpha\) changes from maximum to minimum along the \(z\)-axis in the range \((0 \rightarrow \lambda_g/2)\) and after that the electron executes sinusoidal oscillation. In addition, if the electron is injected at the centre of the waveguide \((x = a/2, y = b/2)\), it will not suffer any deflection at all. However, for simplicity, the electron is assumed to be injected at \(x = a/2\). So, one has

\[
\alpha = \tan^{-1} \left\{ \left( \frac{e\pi}{m_c k_c^2 v_z^2} \right) E_0 \cos\left( \frac{2\pi x}{\lambda_g} \right) \cos(\pi y/b)
\]

\[
\times \left( \frac{\omega e v_z \mu_0}{k_g} - 1 \right) \right\}.
\]  

(11)

Therefore, the distance traveled by the electron along the waveguide can be obtained from Eq. (11), which is expressed in terms of \(\lambda_g\). It is seen that the angle of deflection is directly proportional to \(E_0\), i.e., to the microwave intensity \(I_0\), and it also depends on the waveguide height \(b\) and the microwave frequency \(\omega(= 2\pi f)\).

It is clear that the angle of deflection will be maximal at the front of the waveguide \((z = 0)\) and decreases along the \(z\)-axis to zero at \(z = \lambda_g/4\). However, the angle
of deflection can be analyzed by giving typical values to the waveguide dimensions, the frequency and intensity of the microwave radiation. So, the results can be discussed graphically from the following figures.

From Fig. 1, it can be seen how the deflection angle changes along the z-axis in a rectangular waveguide with a width of 4.0 cm and a height of 2.5 cm for different microwave intensities (a) $2 \times 10^7$ W/m$^2$ and (b) $1 \times 10^8$ W/m$^2$. Here, it is considered that an electron of energy 50 keV is injected at $y = 0.625$ cm. Also the variation in the angle of deflection with microwave intensity along the waveguide is shown in Fig. 2. It is clear that the angle increases for larger microwave intensity and decreases with the increase in distance.

Fig. 1 Change of angle of deflection in degree along the z-axis (m) in a rectangular waveguide with a width of 4.0 cm and a height of 2.5 cm for microwave intensity of (a) $2 \times 10^7$ W/m$^2$ and (b) $1 \times 10^8$ W/m$^2$, when the electron with energy of 50 keV is injected

Fig. 2 Variation in the angle of deflection with the intensity of microwave along the axis of the waveguide. Other parameters are the same as in Fig. 1

The variation in the injection point of the electron on the y-axis of the waveguide upon the deflection angle along the axis of the waveguide at a microwave intensity of $2 \times 10^7$ W/m$^2$ is shown in Fig. 3. It is clearly shown that the deflection angle gets small as the injection point approaches $y = b/2$. Also, it gets smaller as the width of the waveguide gets wider. The effect of both the electron’s initial energy and the frequency of the microwave on the deflection of the electron during its motion along the z-axis is shown in Figs. 4 and 5, respectively. It is revealed that the energy of the injected electron affects slightly the deflection of the electron, while the frequency can decrease the deflection significantly, with the other parameters kept the same as in Fig. 1.

Fig. 3 Variation in deflection angle with the injection point of the electron on the y-axis of the waveguide along the z-axis with a microwave intensity of $2 \times 10^7$ W/m$^2$ and other parameters the same as in Fig. 1

Fig. 4 Effect of the initial energy of the injected electron on the deflection angle during its shift along the z-axis, with a microwave intensity of $2 \times 10^7$ W/m$^2$ and other parameters the same as in Fig. 1

3 Electron acceleration and energy gain

In this section, the electron acceleration during its shift in the field of the TM$_{11}$ mode of microwave radiation along the direction of its propagation is studied. To obtain the acceleration gradient and the energy gain by the electron, the following momentum equation and
energy equation are applied,
\[ \frac{d}{dt}(m_e \gamma v) = -e [E_0 \mu_0 (v \times H)], \quad (12) \]
\[ \frac{d}{dt}(m_e \gamma^2 \zeta) = -e v \cdot E, \quad (13) \]
where energy \( U = m_e \gamma c^2 \) and \( m_e \) is the rest mass of the electron.

As is seen that the velocity \( v_z \) is dominant (i.e. the propagation of the electron is solely specified in the \( z \)-direction), one can transform the coordinates to \( \zeta = v_0 t - z \) to find
\[ m_e (v_y - v_z) (v_\zeta) = -e (E_z - \mu_0 v_z H_y), \quad (14) \]
\[ m_e (v_y - v_z) (v_y) = -e (E_y + \mu_0 v_z H_x), \quad (15) \]
\[ m_e (v_y - v_z) (v_\zeta) = -e [E_z + \mu_0 (v_x H_y - v_y H_z)], \quad (16) \]
\[ m_e c^2 (v_y - v_z) \gamma \zeta = -e [v_x E_x + v_y E_y + v_z E_z], \quad (17) \]
where the subscript \( \zeta \) denotes the differentiation with respect to \( \zeta \).

Eqs. (14) ~ (17) can be solved to obtain the following relation,
\[ [m_e c^2 (v_y - v_z) \gamma \zeta] = -\frac{e^2}{m_e (v_y - v_z)} \times [(E_x^2 + E_y^2) - \mu_0 v_z (E_x H_y - E_y H_z) - e E_z (v_\zeta)]. \quad (18) \]

For the sake of simplicity, it is assumed that both \( \gamma \) and \( v_z \) (in the case of the microwave) are slowly varying (i.e., \( \gamma, v_z \) << \( \gamma, v_z \) \( \zeta << \) \( v_z \zeta \) \( << \) \( v_z \)). Then Eq. (17) becomes
\[ \frac{e^2}{m_e (v_y - v_z)} \times [(E_x^2 + E_y^2) - \mu_0 v_z (E_x H_y - E_y H_z)]. \quad (19) \]

Substituting the value of field components of the mode in Eq. (3) into Eq. (19), one can obtain, by considering \( \zeta \) as a dummy variable,
\[ \left( \frac{d\gamma}{d\zeta} \right)^2 = 2R \ln \gamma + C, \quad (20) \]

where \( C \) is the constant of the integration and
\[ R = \frac{e^2 \pi^2 k^2 E_0^2}{m_e c^2 (v_y - v_z)^2 k_e^2} (1 - \mu_0 v_z \omega \epsilon/k_y) \times \cos(\pi y/b) \sin(2\pi z/\lambda_y), \quad (21) \]
with \( \gamma \) related to the deflection angle \( \alpha \) through
\[ \gamma = c/\sqrt{v^2 - v_z^2 (\tan^2 \alpha + 1)}. \quad (22) \]

As the electron is injected along the \( z \)-axis at the entrance \( (\zeta = 0) \), \( v = v_{0} \) and \( \gamma = \gamma_{0} \), which gives the constant \( C = 2R \ln \gamma_{0} \). Then Eq. (19) becomes
\[ \frac{d\gamma}{d\zeta} = \sqrt{\frac{2}{\pi} \ln \frac{\gamma_{0}}{\gamma_{0}}} \times \cos(\pi y/b) \sin(2\pi z/\lambda_y). \quad (23) \]
Here, \( d/d\zeta = -d/dz \) is used.

Therefore, the change in energy gain \( (U = \gamma m_e c^2) \) per unit distance (eV/m) (i.e. the energy gradient) can be obtained as
\[ \frac{dU}{dz} = \frac{\pi c E_0}{(v_y - v_z) \gamma_{0}^2} \left[ 2 \ln \frac{\gamma_{0}}{\gamma_{0}} \right]^{1/2} \times \cos(\pi y/b) \sin(2\pi z/\lambda_y). \quad (24) \]

It is clear from Eq. (24) that the acceleration gradient depends on the parameters of the microwave, where it increases, for example, with the microwave intensity. It is also indicated that no acceleration will occur as the electron is injected at \( y = b/2 \), while the acceleration gradient has maximal value when the electron completes the distance of \( n\lambda_g/4 \) \( (n = 1, 3, 5, \ldots) \).

Again, we can give typical values of the waveguide dimensions and microwave parameters to analyze the energy gain of the electron during its motion. In Fig. 6 it is shown that the energy gain (acceleration gradient) acquired by the electron varies with microwave intensity along the \( z \)-axis of the waveguide at a microwave frequency 7.5 GHz when the electron is injected with energy of 50 keV. In Fig. 7 the variation of acceleration gradient with frequency of microwave radiation for an intensity of \( 2 \times 10^7 \) W/m² is shown.

Fig.5 Effect of the frequency of the incident microwave on the deflection angle of the electron during its motion along the \( z \)-axis, with the microwave intensity of \( 2 \times 10^7 \) W/m² and other parameters the same as in Fig. 1.
4 Conclusions

The dynamics of an electron in the fields associated with TM\textsubscript{11} modes, excited by microwave propagating along a rectangular waveguide, was investigated. The field components of this mode are calculated and the deflection angle of the electron during its shift along the waveguide was obtained. It is found that the deflection angle depends on the parameters of microwave. By giving typical values for the waveguide dimensions and microwave parameters, it is indicated that the deflection angle increases with the intensity of microwave and the width of waveguide. Also, the deflection angle is reduced as the point of injection of the electron approaches the centre of the waveguide front such that there will be no deflection when it is injected from the centre.

In addition, the acceleration gradient of the electron during its motion in the above mode is studied. It is found that the acceleration gradient increases with the increase in both microwave intensity and frequency. It is also shown that this gradient and hence the energy gain of the electron decrease with the increase in the dimension of the waveguide. However, it may be noted, for example, that the acceleration gradient is enhanced from 0.2 MeV/m to 1.5 MeV/m when the intensity of the microwave increases from $2 \times 10^7$ W/m\textsuperscript{2} to $1 \times 10^8$ W/m\textsuperscript{2} with a frequency of 7.5 GHz and the electron injected in an energy of 50 keV at $y = 0.025$ m.

It is also found that to achieve larger gradients, the electron should be injected with higher energy. Therefore, by optimizing the microwave parameters together with the injected energy and waveguide dimensions, a huge amount of energy gain can be obtained if the proposed mechanism can be used in multiple stages.

References


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