X-Mode Frequency Modulated Density Profile Reflectometer on EAST Tokamak

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Abstract An extraordinary-mode (X-mode) frequency-modulated continuous-wave (FMCW) profile reflectometer has been built on EAST. In the reflectometer, continuous waves with frequency sweeping from 12.5 GHz to 18 GHz were generated through a Hyperabrupt Tuned-varactor Oscillator (HTO) source and a four times active multiplier was used to increase the frequency to V-band (50 GHz to 72 GHz). The polarization of horn lens antenna is perpendicular to the magnetic field line at the edge plasmas. According to the V-band frequency range and polarization, the system cover density range from $0.5 \times 10^{19}$ m$^{-3}$ to $3.0 \times 10^{19}$ m$^{-3}$ (when toroidal magnetic field is 1.8 T), with time resolution of 12.5 to 50 $\mu$s. The density profile could be calculated by assuming the edge profile through an empirical equation. The maximum spatial error deduced by the method is about 4 cm. This reflectometer has been successfully applied in 2010 autumn EAST campaign, the temporal evolution of density profiles was acquired during the low confinement mode to high confinement mode transition. The density pedestal of EAST Tokamak was observed and the top value and gradient of the density pedestal were estimated.

Keywords: reflectometer, electron density, H-mode, CDM

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1 Introduction

Since the first reflectometer was established, microwave reflectometer has become a basic diagnostics in magnetic confinement experiments [1-2]. The principle of microwave reflectometer is based on microwave cut-off phenomena and radar principle. Microwave reflectometer can provide information of electron density profile and electron density fluctuation with high temporal and spatial resolution. Different types of reflectometer systems for density profile measurements have been developed on numerous fusion research devices worldwide [3-8]. The Experimental Advanced Superconducting Tokamak (EAST) [9] is an entirely superconducting tokamak which is similar to the next generation tokamak-International Thermonuclear Experimental Reactor (ITER). In 2010 autumn campaign a solid-state X-mode frequency-modulated (FM) reflectometer was installed and tested on EAST. The frequency range of the reflectometer is from 50 GHz to 72 GHz. It can cover electron density from $0.5 \times 10^{19}$ m$^{-3}$ to $3.0 \times 10^{19}$ m$^{-3}$ when the toroidal field is 1.8 T at the magnetic axis. Another reflectometer from 33 GHz to 50 GHz would be installed in the next campaign. The HTO source has ability of fast frequency sweep [10], after multiplier the frequency of microwave can sweep from 50 GHz to 72 GHz within 10 $\mu$s. This article presents the structure of reflectometer on EAST and experimental result. In remaining part this article is organized as follows: section 2 describes schematic of the reflectometer, profile reconstruct algorithm and profile inverting are presented in section 3, the experimental result will be exhibit in the last section.

2 System schematic and source calibration

Schematic diagram of the reflectometer is shown in Fig. 1. In FM reflectometer, the microwave source generates continuous wave with frequency linearly increasing and HTO was chosen as microwave source for its faster sweeping speed compared with other kinds of sources. Arbitrarily waveform generator (AWG) is used to generate a voltage wave for HTO and it is a standard method for linearizing the sweep of the source frequency by applying a predistorted voltage waveform to correct for the nonlinear tuning of the HTO [10,11]. In the circuit, signal was separated into two channels by a directional coupler. In incident channel, an active 4X multiplier is used to convert low frequency wave to high frequency millimeter microwave. The signal is

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launched into plasma by horn lens antenna which was oriented 90° relative to the edge magnetic field direction. Referenced signal and received signal are mixed by a balanced mixer, then the IF signal is acquired by hi-speed data acquisition cards (DAQ) with 12 bit resolution and a maximum sampling speed of 60 MHz. The coaxial delay line is used to adjust the time difference between the referenced signal and the received signal, namely, adjust the minimum frequency of the IF signal. The minimum is set to 5 MHz to avoid the noises induced by stray reflections from the vacuum chamber. It should be noticed that the transmitting and receiving antennas are located outside the tokamak, it is convenient for distance and phase calibrations. A flat metal plate in front of the antennas was used for calibration. In fact, after the total devices were installed on the tokamak, it was found that the graphite tiles located on the inside wall of the vacuum chamber could also be used as a fixed target mirror.

![Fig.1 Schematic diagram of the reflectometer (color online)](image)

3 Reflectometer analysis and simulation

3.1 Profile reconstruction arithmetic

According to Maxwell equation, the extraordinary dispersion relation in cold plasma approximation can be written as:

$$N^2 = \frac{k^2 c^2}{\omega^2} = 1 - \frac{\omega_{pe}^2}{\omega^2} \frac{\omega^2 - \omega_{pe}^2}{\omega^2}$$  \hspace{1cm} (1)

In the equation, $(\omega, k)$ is the frequency and wavenumber of the microwave, $N$ is refractive index, $\omega_{pe}$ and $\omega_{pe}$ are the electron plasma frequency and upper hybrid resonance frequency. When $N = 0$ microwave can not propagate and cutoff in plasma, so the righthand X-mode cutoff frequency is

$$\omega_R = -\frac{\omega_{pe}}{2} + \sqrt{\omega_{pe}^2 + \frac{\omega_{pe}^2}{4}}.$$  \hspace{1cm} (2)

In microwave reflectometer, the basic information for inferring the plasma density is the phase delay of the reflected wave. The phase delay is given by the equation below:

$$\phi(\omega) = \frac{2\omega}{c} \int_{x_0}^{x_c} N(\omega, x)dx + \phi_0(\omega),$$  \hspace{1cm} (3)

where $x_0$ and $x_c$ are the coordinates of the plasma boundary and the cutoff layer, and $\phi_0(\omega)$ is the phase delay that the wave suffers outside the plasma and could be calculated independently. In the following $\phi_0(\omega)$ will not be mentioned for simplify. For the X-mode, the first term on the right-hand side of Eq. (3) should be calculated numerically according to the following definition [2]:

$$\phi_i = \Sigma_{j=1}^{i} A_{i,j} (x_j - x_{j-1}),$$

$$A_{i,j} = \omega_i (N_{i,j} + N_{i,j-1})/2c \quad 1 \leq j \leq i,$$  \hspace{1cm} (4)

where $x_j$ is the cutoff position of the frequency $\omega_j$.

In data processing program, short time Fourier transform is used to calculate frequency of IF signal $f_{IF}$. Time delay $\tau$ of RF signal and phase change $\phi_i$ is extracted by:

$$\tau = d\phi/d\omega = \frac{d\phi/dt}{d\omega/dt} = f_{IF} \frac{df/dt}{d\omega/dt},$$

$$\phi_i = \sum_{j=1}^{i} \tau_j (\omega_j - \omega_{j-1}).$$  \hspace{1cm} (6)

So, constant sweeping rate $df/dt$ could make sure that $\tau \propto f_{IF}$ and $\tau$ could be calculated easily. The profile inverting program can be written in matrix form [1]. In the equation $M$ is a parameter which is related to refractive index.

$$\phi = M \cdot x,$$  \hspace{1cm} (7)

$$M = \begin{bmatrix} A_{1,1} & 0 & 0 & \cdots \\ A_{2,1} - A_{2,2} & A_{2,2} & 0 & \cdots \\ A_{3,1} - A_{3,2} & A_{3,2} - A_{3,3} & A_{3,3} & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ A_{n,1} - A_{n,2} & A_{n,2} - A_{n,3} & A_{n,3} - A_{n,4} & \cdots \end{bmatrix}$$  \hspace{1cm} (8)

$$A_{i,j} = \omega_i (N_{i,j} - N_{i,j-1})/2c.$$  \hspace{1cm} (9)

$N_{i,j}$ is the refractive index at location $x_j$ for microwave with frequency $\omega_i$.

The density profile can thus be evaluated in a step by step manner using the above equations. It could be seen that an accurate phase delay series $\phi_i$ is the basis of the reconstruction and the key aspect is the initialization, i.e. the information of the zero density position [7,12]. However, the reflectometer frequency range
is from 50 GHz to 72 GHz here and the plasma boundary is beyond the measurement range. The edge profile must be modeled and the details would be described as follows. This method has been used before for DIII-D reflectometer [4].

### 3.2 Extracting the phase information

For our system, the phase delay $\phi_i$ curve can be readily extracted from the spectrogram by taking the time evolution of the main spectral peak $f_{IF}$ in each time window. The time-frequency distribution provided a good insight into the frequency content of the reflectometer signal along the frequency sweep and the deviation of $f_{IF}$ from the ideal value (geometric optical approximation, no dispersion plasma) is mainly caused by the scattering of the signal energy to frequencies due to turbulence effects. In our analysis, the signals which are strongly interfered by the turbulence were directly abnegated manually because automatic profile evaluation is still not needed due to the missing of Q-band (33–50 GHz) reflectometer. It is found that the percentage of the abnegated signals is less than 5% and the regular signals show high energy in a narrow frequency band ($\Delta f < 1.5$ MHz), which are adaptive to the complex demodulation technique [13]. Fig. 2 shows the typical regular time-frequency distribution and the abandoned distribution which is distorted by the turbulence.

![Fig.2](image)

Fig.2 The time-frequency spectrum of (a) the regular signal and (b) the distorted signal (color online)

The time-frequency spectra in Fig. 2 are calculated through short-time Fourier transform (STFT) and the major drawback of STFT is the deviation induced by the trade-off between time and frequency resolution [13]. In our system, for example, the IF signal in one sweep time of 20 $\mu$s is sampled with Nyquist frequency $F_s=60$ MHz. Such a time series with 1200 points is divided into 51 time windows with 200 points each and 180 points overlapped, which means the first and the last $f_{IF}$ values are calculated through the first and the last 200 numbers respectively. It is naturally to regard the first $f_{IF}$ as IF frequency at $t = 0$ and the last $f_{IF}$ as that at $t = 20$ $\mu$s, which would induce notable deviation of the phase delay. An alternative method is the digital complex demodulation (CDM), which is essentially a software implementation of analog heterodyne demodulation. The principle of CDM technique is described in Ref. [14] and it is used as a regular method in DIII-D profile measurements [15]. A detailed comparison would be given between the two approaches on a simulated signal.

Firstly a cosine time series with time interval $t = 20$ $\mu$s, sampling frequency $F_s=60$ MHz is created as the IF signal and $f_{IF}$ varied from 6 MHz to 17 MHz linearly. These parameters are similar to that of the actual signal. Then the phase delay $\phi_{ideal}(t) = 2\pi f_{IF} t$ could be calculated exactly. In Fig. 3(a) the time-frequency spectrum through STFT is shown. The phase series could then be calculated through STFT with different window length and CDM, and the deviations between the calculated phase delays and $\phi_{ideal}(t)$ are shown in Fig. 3(b). The $nfft$ values in Fig. 3(b) means the window length. It could be seen that for an ideal signal without noise, the accuracy of CDM is better than STFT since CDM is a quadrature technique and the demodulation is obtained through filtering and time-window dividing is not needed [14].

![Fig.3](image)

Secondly, the amplitude noises and phase noises are added to the ideal signal through a factor to adjust the signal to noise ratio:

$$
x_1(t) = (1 + \alpha G(t)) \cos(2\pi f(t)), 
$$

$$
x_2(t) = \cos(2\pi f(t) + \beta G(t)), 
$$

where $G(t)$ is Gaussian noise and $f(t)$ is still changed linearly from 6 MHz to 17 MHz. Similar to the above, phase are calculated from $x_1(t)$ and $x_2(t)$ through STFT ($nfft = 58$ is used) and CDM approaches and the deviations from ideal values are compared as $\alpha$ and $\beta$ varying from 0 to 1. The results are shown in Fig. 4. It can be seen that when $\alpha, \beta < 0.6$ the accuracy of CDM is still much better than STFT and signal to noise ratio has little effect on STFT results. The nice resistance to moderate noise of CDM is because the CDM results are calculated through accumulation on the complex phase of a plural series (the so-called complex demodulation) and the phase could be extracted insensitive to amplitude modulation [14]. So it is easy to understand that large noise, either amplitude noise or phase noise, would force the plural series to cross zero and cause spurious phase jumps. It is why the deviation of CDM in Fig. 4 changes like stairs. The noise test implies that CDM approach could avoid the
drawback of Fourier transform and it is applicable to most data with moderate noise. In practice, the two methods are combined together in the data analysis, because the range of the filters is determined through time-frequency spectrum and the choice of appropriate filters is crucial for the performance of CDM.

3.3 Simulated density profile

As mentioned in the previous subsections, one of the advantages of using X-mode reflectometer is that the edge plasma density could be measured. The radial position where IF signal change at the start of reflection from right-hand cutoff frequency is called plasma start location and the profile inversion should start from this location, However, in this experimental campaign on EAST only V-band data were acquired and the plasma start location could not be calculated. So, the edge density profile should be assumed firstly.

A piecewise function was used to simulate the density profiles in \( r/a = 0 \sim 1.5 \) and three formulas correspond to the core, edge and scrape off layer (SOL) plasmas respectively, as shown below.

\[
y(r) = \begin{cases} 
  n_{\text{max}} \exp(-r^2 + \delta/a^2) & \text{core}(0 \leq r/a < 0.9) \\
  n_{\text{max}} \varepsilon \arctan(1 - r/a) & \text{edge}(0.7 \leq r/a < 0.97) \\
  n_{\text{sep}} \exp[-\eta(r-a)] & \text{SOL}(r/a > 0.95)
\end{cases}
\]  

(11)

There are five parameters in the function, \( n_{\text{max}}, n_{\text{sep}}, \delta, \varepsilon \) and \( \eta \), to describe the maximum density, the separatrix density, downward trend of the core density, steepness of the edge density and the decay length of the SOL density respectively. The three curves described three regions from the maximum to the zero density position and the boundaries of each region were determined by points of intersection between the curves. The mode could describe the main characteristics of most density profiles under L-mode and H-mode, except some special cases like the hollow distribution and internal transport barrier situation. Fig. 5 shows two normalized profiles, \( [\delta, \varepsilon, \eta] = [0.2, 2.2, 20] \) and \( [\delta, \varepsilon, \eta] = [1, 5, 50] \) correspond to L-mode and H-mode distributions respectively. Based on the assumed profile, the simulated IF signals including both Q-band and V-band could be calculated and the simulated \( f_{\text{IF}} \) curve could be obtained through STFT.

The criterion to choose the suitable profile is the consistency between the simulated \( f_{\text{IF}} \) curve and the actual one, especially the continuity at \( f_{\text{inj}} = 50 \text{ GHz} \) if the simulated Q-band curve and the actual curve are linked together. Fig. 6 shows how two simulated and one actual IF frequency curves change with the injected microwave frequencies. The blue solid line and red dash line in Fig. 6 correspond to the profiles in Fig. 5 respectively and the black star is derived from 20 \( \mu \)s data in shot 33068. It should be announced that before calculating the simulated IF signal, the reflecting layer of injected 30 GHz microwave is chosen as the plasma start location artificially, which could only be smaller than the actual start frequency on EAST based on the experimental results on other devices \([1, 15]\). So the peaks at \( f_{\text{inj}} \sim 36 \text{ GHz} \), as could be seen in Fig. 6, were caused by the numerical calculation and probably not exist in actual Q-band signals. The consistency of the red dash
3.4 Spatial error analysis

It is difficult to exactly calculate the spatial error of the density profile without the real Q-band signal. One feasible approach is comparing the final profiles under various available parameter combinations. A rough quantitative result could be given by this time-consuming testing. Firstly, a threshold of the total deviation between the assumptive and the acceptable ranges of parameters was chosen deliberately to compare the maximal difference between the reconstructed profiles. Here an example is shown in Fig. 7. In Fig. 7(a) three IF frequency curves are given and the simulated curves of two groups of parameters \(\delta \epsilon = [-0.05, 0.05], [-0.2, 0.2]\) show acceptable consistency with the actual one. However, the continuity at \(f_{\text{inj}}=50\) GHz is not very well, and \(f_{\text{IF}}\) at 50 GHz in one curve is a little larger than the actual value and the other is a little smaller. The two groups of extreme parameters are chosen deliberately to compare the maximal difference between the reconstructed profiles, as shown in Fig. 7(b), where each density profile consists of solid line and dashed line which respectively represent the actual data and simulated data. The results of comparison show that the radial displacements between the density profiles deduced from the same V-band data combined with various Q-band data are mostly around 2 cm and the maximum is less than 4 cm, which could be used as a rough estimate of the radial error of our density reconstruction.

![Fig. 7](image)

\(\delta \epsilon = 0.05 \times 5 \eta = 20\)
\(\delta \epsilon = 0.2 \times 2 \eta = 20\)

4 Experimental result

Based on the simulation and arithmetic above, some preliminary experimental results on EAST density profiles would be given. The first H-mode with an H factor of \(H_{\text{IPB98}(2)} \sim 1\) has been obtained with about 1 MW lower hybrid wave power and lithium-wall coatings in 2010 autumn campaign on EAST \((R_0=1.88\text{ m}, a=0.45\text{ m}, B_t < 3.5\text{ T}, I_p \sim 1\text{ MA})\) [17]. The density profile evolution was recorded by the reflectometer. An example is shown in Fig. 8, where four profiles at different moments during the L-H transition are given: typical L-mode, a few milliseconds after H-mode, a few milliseconds before the first edge localized mode (ELM) and after lots of ELMs. These moments could be identified in Fig. 8(a) which shows the \(D_t\) emission signal. The density profiles in Fig. 8(b) clearly demonstrate the pedestal formation at the onset of H-mode, and then the pedestal width increased sustainedly with almost constant slope until the first ELM occurred, and after the type-III ELMs lasting about 100 ms, the pedestal slope still changed little at 2900 ms, implying that ELMs only degraded the pedestal height. The time-frequency spectrum of IF signal could also imply the change of density gradient, as shown in the bottom of Fig. 8. In the lab experiment, mirror target would generate a constant IF value, so the similar flat IF curves in the time-frequency spectra of \(t=2640\text{ ms}, 2709\text{ ms}, 2900\text{ ms}\) imply steep density profiles, and the size of the platform region is determined by the radial range of the steep profiles. From these profiles, it could be calculated that before the ELMs emerging, the maximum pedestal width is about 4 cm and the pedestal top is about \(3.0 \times 10^{19} \text{ m}^{-3}\) in shot 33068. Another example is given in Fig. 9, showing the profile change from H-mode to L-mode in three moments. Both the profiles and the time-frequency spectra show that during the transition the main change of pedestal is the height and the width.

![Fig. 8](image)

(a) \(D_t\) emission signal, (b) The density profiles and (c1~4) STFT spectra of IF signals during L-H transition at shot 33068. The dash lines refer to the simulated Q-band profiles (color online)
Fig. 9  (a) $D_\alpha$ emission signal, (b) The density profiles and (c1~4) STFT spectra of IF signals during H-L transition at shot 35764. The dash lines refer to the simulated Q-band profiles (color online).

5 Conclusion and future work

In summary, the X-mode frequency-modulated reflectometer has been firstly applied to EAST. Through certain assumptions on the edge profile, the density profiles with high time resolution could be estimated and the pedestal evolutions have been recorded. Some hardware improvements will be done in the next season: firstly, the antennas will be placed in the vacuum chamber linking with over-mode waveguides to reduce the spurious scattering and improve the signal-to-noise ratio; secondly, microwave frequency coverage will be extended to Q-band to confirm the plasma start position and profile assumptions are not needed any more; thirdly, data acquisition system will be replaced by a continuous acquisition system including a data streaming system with 700 MB/s of sustained data read and write rates and DAQ cards that can stream data at the full data rate of 400 MB/s to ensure uninterrupted acquisition during a discharge. The data processing technique should also be improved for automatic profile evaluation in future.

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References


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