A Particle-in-Cell Simulation of Double Layers and Ion-Acoustic Waves

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Abstract Double layers and ion-acoustic waves are investigated by using a one-dimensional electrostatic particle-in-cell simulation code. Our results show that double layers can be formed even when the drift velocity between electrons and ions is less than the electron thermal velocity. Electron and ion density depressions were clearly seen. Electrons gradually developed a distribution comprising both background and beam components. In fact, as the initial electron-ion drift velocity was less than the electron thermal velocity, intense ion-acoustic waves could be found only at the places where the electron beam was located, suggesting that they are excited by the self-consistently developed electron beam. Besides the Langmuir waves and ion-acoustic waves, the beam mode excited by electron beams produced in our simulation has been clearly found.

Keywords: electron-ion streaming instability, Langmuir instability, particle-in-cell

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1 Introduction

Double layers have long been of interest to both laboratory and space plasma communities \cite{1,2,3,4,5,6,7}. Definitive double layer structures were observed by the Viking satellite in the magnetosphere, which suggests that double layers are small. These structures have an extent of $\sim 100$ m, amounting to a few tens of Debye lengths, and the plasma density therein is reduced by up to 50\% \cite{8}. The necessary condition for the formation of double layers obtained from the previous works \cite{1,2,3,4} is that the electron drift velocity exceeds the electron thermal velocity, i.e., double layers are a result of the Buneman instability. If the opposite is true, i.e., the relative streaming velocity is sufficiently less than the electron thermal speed, the instability is called the ion-acoustic (IA) instability. Both the Buneman \cite{9,10,11,12} and IA instabilities \cite{13,14,15} have been investigated carefully. However, there have been observations \cite{16,17} that support the existence of double layers along auroral field lines where the electron drift speed is much less than the electron thermal speed. SATO and OKUDA \cite{18} pointed out that an IA instability could result in the formation of double layers if a system was sufficiently long. The initial drift velocity used in their simulation was less than the electron thermal velocity. They also pointed out that the anomalous resistivity generated by the IA instability led to the buildup of a DC potential which in turn further accelerated electrons to enhance the original instability, leading to the formation of double layers. But a low ion-to-electron mass ratio (which is equal to 100) was used in their simulation. What consequences will a realistic mass ratio (1836) bring forth?

Computer simulations are the most powerful tool to study this dynamical process. For instance, Vlasov simulations \cite{13,14,15,19,20,21,22,23} have been used to study the electron dynamics, and the anomalous resistivity produced therein via the IA instability. Many authors used one- or two-dimensional particle-in-cell codes to examine electrostatic instabilities \cite{24,25,26,27}. To explore the formation of double layers in general, in particular the wave and particle dynamics, we use a 1D treatment in this study to help isolate the physics we are interested in, by excluding complications such as oblique wave modes.

In this paper, using one-dimensional electrostatic particle simulation, we will present the results of kinetic simulations designed to study the development and evolution of double layers in a current-carrying plasma. A relatively low velocity $u_0/v_{\text{the}}=0.6$ is used. The excitation and evolution of ion-acoustic waves will also be discussed.

2 Simulation

We have performed one-dimensional electrostatic particle simulation with system length $L=1024\lambda_D$, where $\lambda_D=v_{\text{the}}/\omega_{pe}$ is the electron Debye length, $v_{\text{the}}=$...
Along the drift direction, electrons are decelerated and then accelerated in the whirl region. On the contrary, ions are accelerated and then decelerated. The figure also suggests that the number of energetic electrons decreases while the number of energetic ions increases in the whirl region. The electric field with a bipolar structure corresponds to the ion phase-space holes. It is therefore clear that double layers can form through the ion-acoustic instability. In fact, the biggest whirl consists of two double layers.

**3 Simulation results**

Fig. 1(a) shows the electron drift speed $u_e$ versus normalized time. Fig. 1(b) shows the evolution of electric energy density. In the early stage of wave evolution ($\omega_{pe}t=0$–2500) the total field energy increases almost linearly with time and the electron drift speed $u_e$ drops slowly at a roughly constant rate. A drastic reduction of electron drift speed, and hence a sudden enhancement of the total field energy takes place at $\omega_{pe}t\approx 2700$. By the end of the simulation $u_e$ approaches $\sim 0.43v_{the}$. Both the electric energy and $u_e$ undergo some significant overshoot around $\omega_{pe}t=5200$ or so.

Fig. 2(a) and (c) show the electron and ion phase-space distributions, with the red solid lines representing $v/v_{the}=0$; Fig. 2(b) and (d) show the electron and ion density profiles along $x$, with the red solid lines representing $n/n_0=1$; Fig. 2(e) shows the electric field distribution at $\omega_{pe}t=4900$ when the double layer has fully developed. A density depression, $\Delta n/n_0 \sim 0.55$, is generated for both electrons and ions at about $x/\lambda_D \sim 230$. In Fig. 2(c), the ion phase-space holes (or whirls) are obvious. Ion phase-space holes have been found in previous simulations \cite{6, 7}, but a relative drift velocity that is larger than the electron thermal velocity was used there. Here, we mainly discuss the biggest whirl region. Along the drift direction, electrons are decelerated and then accelerated in the whirl region. On the contrary, ions are accelerated and then decelerated. The figure also suggests that the number of energetic electrons decreases while the number of energetic ions increases in the whirl region. The electric field with a bipolar structure corresponds to the ion phase-space holes. It is therefore clear that double layers can form through the ion-acoustic instability. In fact, the biggest whirl consists of two double layers.

**Fig.1** (a) The electron drift speed $u_e$ versus normalized time, (b) The evolution of electric energy density $E^2$

**Fig.2** (a) and (c) the electron and ion phase-space distributions, (b) and (d) the electron and ion density profiles, (e) the electric field distribution. All results are for $\omega_{pe}t=4900$ (color online)

Fig. 3 plots the velocity distributions of electrons and ions at two different instants. The black lines give the distributions at $\omega_{pe}t=0$ and the red lines are for those at $\omega_{pe}t=4900$. The electron velocity distribution has undergone some substantial change from the initial drifting Maxwellian by $\omega_{pe}t=4900$. The value of
electron $f(v)$ decreases at lower positive phase velocities and increases at lower negative phase velocities, meaning that electrons with a small positive velocity tend to be trapped by the ion-acoustic instability. An electron beam is formed at $\omega_{pe} t = 4900$, which is consistent with the previous results [6]. This distribution seems to suggest that the ion-acoustic instability only affects electrons with small speeds, which is different from the simulation results of the Buneman instability. The modulation of ion distribution shows that ions are accelerated and heated. A comparison with Fig. 2 suggests that ions are accelerated in the whirls.

Fig. 3 The velocity distributions of electrons and ions at $\omega_{pe} t = 0$ (the black curves) and $\omega_{pe} t = 4900$ (the red curves) (color online)

Fig. 4 shows the time histories of (a) electron density $n_e/n_0$, (b) ion density $n_i/n_0$, (c) electron drift speed $u_e/v_{the}$, (d) ion drift speed $u_i/v_{the}$, (e) electric field, and (f) electrostatic potential $e\phi(x,t)/T_e$. The electron and ion densities show a depression of up to 55%. Fig. 4(c) shows that for the majority of grid points, the electron drift speed gradually decreases, in much the same way as the one averaged over the simulation domain shown in Fig. 1. Something different appears nonetheless. At some points, corresponding to the ridges in the $x - t$ plane, the drift speed increases instead. In fact, the velocity distributions can be seen as comprising both background and beam electrons. The maximum value is much larger than the initial drift velocity $u_e/v_{the}=0.6$. Fig. 4(d) also shows clearly a ridge, around which the ion drift speed increases substantially, albeit directed in the opposite direction. Fig. 4(e) and 4(f) show an obvious double layer, which is actually composed of several double layers. The most intense one can be seen near $\omega_{pe} t \sim 4900$. The maximum and minimum values of the dimensionless electric field are 0.33 and $-0.22$, respectively. This localized structure propagates along the positive $x$ direction, and its width is about thirty Debye lengths, consistent with observational results [1,8]. Compared with the phase-space holes shown in Fig. 2, the electric field with a bipolar structure is strong and obvious in the region where the whirls are most evident.

Fig. 4 The evolutionary history of (a) electron density $n_e/n_0$, (b) ion density $n_i/n_0$, (c) electron drift speed $u_e/v_{the}$, (d) ion drift speed $u_i/v_{the}$, (e) electric field and (f) electrostatic potential $e\phi(x,t)/T_e$ (color online)

To examine the time evolution and the excitation of ion-acoustic waves, we apply a wavelet analysis to the time series of the electric field. Fig. 5(a1), (b1), (c1) and (d1) present the wavelet power spectrum at $x/\lambda_D=100$, 170, 210 and 230, respectively. Here the vertical and horizontal axes denote frequency and time, respectively. In addition, the thick contour encloses regions where the confidence level exceeds 95%. The horizontal dashed lines correspond to the electron and ion plasma frequencies, $\omega_{pe}$ and $\omega_{pi}$, respectively. The area below the thick dotted contour indicates the cone of influence where edge effects become important. Fig. 5(a2), (b2), (c2) and (d2) present the drift velocity evolutions at the corresponding locations.

The ion-acoustic waves shown in Fig. 5(a1) are very weak, while the electron drift velocity shown in (a2) shows a rather modest decrease at a roughly constant rate during the entire simulation. In Fig. 5(b1), the ion-acoustic waves are noticeable from $\omega_{pe} t \sim 1200$ to $\sim 2500$. The dominant frequency of the wave is $\omega/\omega_{pe} \sim 0.019$, which is below the ion plasma frequency. Fig. 5(b2) suggests that the electron drift velocity increases somehow at $\omega_{pe} t \sim 1200$ and then decreases. At $x/\lambda_D=210$, shown in Fig. 5(c1), the waves are significantly intensified. Moreover, the wave power seems to be concentrated at two frequencies, $\omega/\omega_{pe} \sim 0.011$ and $\omega/\omega_{pe} \sim 0.019$, during the interval between $\omega_{pe} t \sim 3500$ and $\sim 4600$. The corresponding relative drift velocity shown in Fig. 5(c2) appears to possess two peaks, one
The relative drift velocity will decrease quickly and is about 0.76$v_{th}$ at $\omega_{pe}t\sim3700$, the other being 0.9$v_{th}$ at $\omega_{pe}t\sim3800$. This suggests that ion-acoustic waves are excited by the electron beam. At $x/\lambda_D=230$, as shown in Fig. 5(d), the spectra of the excited waves become even broader. Correspondingly, the relative drift velocity now possesses one obvious peak. In addition to the ion-acoustic waves, some Langmuir waves (LWs) can also be seen, albeit much weaker.

The wave behavior is further examined in Fig. 6, where the $\omega$-$k$ diagram is found by Fourier transforming the electric field. Here frequency is plotted versus wave intensity with the reddish regions indicating the most intense. The result is divided into four time intervals (a) $\omega_{pe}t=0$ to 409.6, (b) $\omega_{pe}t=1800$ to 2209.6, (c) $\omega_{pe}t=3600$ to 4009.6 and (d) $\omega_{pe}t=4800$ to 5209.6. Only LWs at the fundamental are found in Fig. 6(a). In Fig. 6(b), the ion-acoustic instability is visible. As time proceeds, this low-frequency instability becomes intenser and its spectrum gets broader, which can be seen clearly in Fig. 6(c) and (d). In addition to the LWs and IAs, some beam-driven waves can also be found in Fig. 6(c) and (d), as a result of the electron beam shown in Fig. 4(c). And they span a broad spectrum where the phase speed varies little with wave number. This is understandable, given that when the beam density is much less than the total density, the dispersion relation of the beam-driven waves can be approximated as $\omega \approx ku_d$, with $u_d$ being the electron beam drift velocity $^{[27]}$. The slope of the ridge shown in Fig. 6(c) is about 0.9$v_{th}$, which is consistent with the results shown in Fig. 5(c) and (d). Also, the ion-acoustic waves are much stronger than the beam-driven waves. When the amplitude of the ion-acoustic wave increases, the beam-driven waves will become less intense.

**4 Conclusion and discussion**

In this paper, the evolution of double layers and ion-acoustic waves is examined by using a one-dimensional electrostatic particle-in-cell code. The simulation results show that double layers can form even if the initial drift velocity between electrons and ions is less than the electron thermal velocity, which is consistent with previous results $^{[18]}$. However, the double layers in the simulation with a low ion-to-electron mass ratio close to the realistic one is used here. The double layers in our simulation are several tens of Debye-lengths wide, in agreement with the observational results $^{[6]}$. The number of ions with high velocities in the whirls region exceeds that in the rest of the simulation domain. On the contrary, the number of electrons with high velocities in the whirls region is the smallest in the simulated region.

The excitation of ion-acoustic waves is also discussed. First, ion-acoustic waves are localized when the drift velocity between electrons and ions exceeds that in the rest of the simulation domain. On the contrary, the number of electrons with high velocities in the whirls region is the smallest in the simulated region.
tion due to wave particle interactions associated with energy exchange \cite{28,29}. Third, both LWs and beam-driven waves are found in the simulation, but they are much weaker than the ion-acoustic ones.

We also examined the case where a low proton-to-electron mass ratio of 100 is used. This is the ratio adopted in Ref. \cite{18}, and our computations agree closely with the results therein. As expected, in this case the IA waves appear earlier as a result of the lower mass ratio. In particular, the evolution of the electric field is nearly the same as that reported in Ref. \cite{18}, which suggests that an arbitrary, small mass ratio speeds up the development of instability. From this comparison we conclude that our results with a realistic mass ratio are genuine and physical. However, even though we get some interesting results, there are still many questions to be addressed. For instance, one may ask what new wave modes will appear if a finite magnetic field is introduced, in which case the gyromotions of charged particles will certainly play a role. Another direction that is worth pursuing is to adopt the observed values as physical parameters to drive our code. This “data-driven” approach is, however, beyond the scope of the present manuscript.

References


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