The Nucleon-Actinide Global Optical Model Potential Below 300 MeV*

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Abstract A set of new global phenomenological optical model potential parameters has been obtained in the mass range of target nuclei 220 ≤ A ≤ 260 with incident energies below 300 MeV, by simultaneously fitting the experimental data of 232Th and 238U, and these potential parameters are analyzed and used to calculate the reaction cross sections, energy spectra and double differential cross sections for p+-232Th reaction. Comparison of calculated results using these potential parameters with available experimental data shows that the present form of global optical model potential could reproduce experimental data for both the neutron and the proton.

Keywords: global optical model potentials, fission nuclear reaction, nucleon


DOI: 10.1088/1009-0630/14/6/14

1 Introduction

The optical model is one of the most fundamental theoretical models in nuclear reaction theory. It has a significant impact on many branches of nuclear reaction physics. The key point of the optical model is its potential. The crucial assumption regarding the model is that the complicated interaction between an incident particle and a nucleus can be represented by a complex mean-field potential, which divides the reaction flux into a part covering shape elastic scattering and a part describing all competing non-elastic channels. Solving the Schrödinger equation with this complex potential yields a prediction for the basic observable, namely the elastic scattering angular distribution and total reaction cross sections. The phenomenological optical model potential includes some adjustable parameters, and it is obtained by adjusting its parameters to fit the existing experimental data. The quality of several derived quantities that are provided by the optical model has an important impact on the evaluation of various nonelastic channels. Well-known examples are the related transmission coefficients that enter the statistical model of compound nucleus evaporation, and the distorted wave functions that are used for the description of direct inelastic scattering to discrete states as well as in evaluations of multi-step direct transitions to the continuum. Therefore, it is crucial that the optical model potentials that enter such nuclear model calculations are adequately determined. A global optical model, in which a potential is specified for both a mass region and an energy region, and an important feature of the global optical model potentials is that it can be used to reliably predict these observable for energies and nuclei for which no experimental measurement data exist. Furthermore, the optical model potential plays a key role in nuclear astrophysics. Reliable optical model parameters are important in the determination of reaction cross sections and rates for astrophysics.

In addition, knowledge of accurate cross sections of a number of reactions (e.g., total, nonelastic, fission) between neutrons and actinides is crucially important for designing various reactor systems. In the accelerator-driven system (ADS) of radioactive waste transmutation and energy generation, the nuclear reaction data are needed for both neutrons and protons as projectiles up to several hundred MeV. The optical model is one of the fundamental theoretical tools that provide the basis for nuclear reaction data analysis and various cross sections.

KONING and DELAROCHE[1] gave phenomenological local and global optical model potentials for neutrons and protons with incident energies from 1 keV up to 200 MeV for (near-)spherical nuclides in the mass range 24 ≤ A ≤ 209, in which the appropriate experimental data were available. These potentials are based on a smooth, unique functional form for the energy dependence of the potential depths on physically constrained geometry parameters.

In the present paper, a set of nucleon phenomenological global optical model potential parameters for the actinide in the charge number range 89 ≤ Z ≤ 100 and the mass range 220 ≤ A ≤ 260 is obtained based on the appropriate experimental data, including the neutron total cross sections, nonelastic cross sections, elastic scattering cross sections, elastic scattering angular distri-
buttions, and proton reaction cross sections and elastic scattering angular distributions of $^{232}\text{Th}$ and $^{238}\text{U}$ with incident nucleon energies below 300 MeV. Moreover, we also calculate the reaction cross sections, energy spectra and double differential cross sections of neutron, proton, deuteron, triton, and alpha-particle emission for $p^+^{232}\text{Th}$ reactions by the obtained nucleon global optical model potential parameters.

2 Theoretical models

2.1 The optical model potential

The optical model potentials considered here are Woods-Saxon[2] form for the real part; Woods-Saxon and derivative Woods-Saxon form for the imaginary part. The analytical expression of the phenomenological optical model potential form is

$$V(r, E) = V_R(r, E) + i[W_S(r, E) + W_V(r, E)] + V_C(r),$$

(1)

The energy dependence of potential depth is given by

$$V_R(E) = V_0 + V_1 E + V_2 E^2 + V_3 \left(\frac{N-Z}{A}\right),$$

(2)

$$W_S(E) = \max\{0, W_0 + W_1 E + W_2 (\frac{N-Z}{A})\},$$

(3)

$$W_V(E) = \max\{0, U_0 + U_1 E + U_2 E^2\}. $$

(4)

The radii are given by

$$R_i = r_i A^{\frac{2}{3}}, \quad i = R, S, V, SO, C.$$

(5)

Where $r_R, r_S, r_V, r_{SO}$ and $r_C$ are the radius of the real part, the surface absorption, the volume absorption, the spin-orbit couple and the Coulomb potential, respectively. $a_R, a_S, a_V$ and $a_{SO}$ are the width of the real part, the surface absorption, the volume absorption and the spin-orbit couple potential, respectively.

The units of the potential $V_R, W_S, W_V, V_{SO}$ are in MeV, the lengths $r_R, r_S, r_V, r_{SO}, r_C, a_R, a_S, a_V$ and $a_{SO}$ are in fm, the energy $E$ is in MeV.

The 20 parameters $V_0, V_1, V_2, V_3, W_0, W_1, W_2, U_0, U_1, U_2, V_{SO}, r_R, r_S, r_V, r_{SO}, r_C, a_R, a_S, a_V$ and $a_{SO}$ can be adjusted. Among them, $V_3, W_2, V_{SO}, r_{SO}, r_C$ and $a_{SO}$ are fixed, which are taken from Ref. [2]. The 14 parameters are adjusted in the present work.

Starting with the experimental data of total, nonelastic, elastic cross sections and elastic scattering angular distributions for $^{232}\text{Th}$ and $^{238}\text{U}$, we performed the analysis so as to extract a set of global parameter based on as many experimental data as possible, in which the nucleus of mass range is $220 \leq A \leq 260$ and the incident nucleon energy range is from 1 keV to 300 MeV. The optical model potential parameters obtained are listed in Table 1.

<table>
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<tr>
<th>$V_0$</th>
<th>$V_{SO}$</th>
<th>$r_R$</th>
<th>$r_S$</th>
<th>$r_V$</th>
<th>$r_{SO}$</th>
<th>$r_C$</th>
<th>$a_R$</th>
<th>$a_S$</th>
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<th>$a_{SO}$</th>
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<td>1.2683750</td>
<td>0.00019246</td>
<td>1.14190519</td>
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<td>1.35801959</td>
<td>9.54198456</td>
<td>r$_{SO}$</td>
<td>1.10</td>
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<tr>
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<td>W$_2$</td>
<td>W$_0$</td>
<td>W$_1$</td>
<td>W$_2$</td>
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<td>W$_0$</td>
<td>W$_1$</td>
<td>W$_2$</td>
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<td>-0.00012181</td>
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The radial dependence of the real and imaginary part of the nucleon global optical potential is calculated for $^{238}\text{U}$ at the incident energies of 5 MeV, 20 MeV, 50 MeV, 100 MeV, 150 MeV, 200 MeV and 300 MeV. We can see that the absolute value of the real part decreases with increasing incident energy. The absolute value of the imaginary part of surface absorption decreases and the volume absorption increases with increasing incident neutron energy. The imaginary part is almost wholly contributed by the volume absorption at the incident energy of 200 MeV. These results are given in Figs. 1 and 2.

![Fig.1](image1.png)  
**Fig.1** Radial dependence of the real part of nucleon global optical potential for $^{238}\text{U}$ with different incident neutron energies

![Fig.2](image2.png)  
**Fig.2** Radial dependence of the imaginary part of nucleon global optical potential for $^{238}\text{U}$ for different incident neutron energies

Another important quantity in the study of optical model potentials is the volume integral of the potential, which is also calculated. The volume integral per
The nucleon-actinide global optical model potential below 300 MeV

The nucleon of the optical model potential for nucleon is expressed as

\[ J_V = -\frac{1}{A_p A} \int V(E, r) dr, \]  
\[ J_W = -\frac{1}{A_p A} \int [W_S(E, r) + W_V(E, r)] dr. \]

Here \( A_p \) is the mass number for the projectile.

The volume integrals per nucleon for our phenomenological nucleon global optical potential of real part and imaginary part for \( ^{238}\text{U} \) are given in Figs. 3 and 4. We can see that the volume integral per nucleon of the real part \( J_V \) decreases as the incident energy increases. The volume integral per nucleon of the surface absorption \( J_{W_S} \) decreases as the incident energy increases, and the volume absorption \( J_{W_V} \) increases as the incident energy increases.

\[ \text{Fig. 3} \quad \text{The volume integral per nucleon of the real part of nucleon global optical potential for } ^{238}\text{U} \text{ with different incident neutron energies} \]

\[ \text{Fig. 4} \quad \text{The volume integral per nucleon of the imaginary part of nucleon global optical potential for } ^{238}\text{U} \text{ with different incident neutron energies} \]

### 2.2 The other theoretical models

The preequilibrium statistical theory based on exciton model [3], evaporation models and Hauser-Feshbach theory with width fluctuation correction, and intranuclear cascade model are used to describe the nuclear reaction preequilibrium and equilibrium decay processes for incident nucleon energies below 300 MeV. The improved Iwamoto-Harada model [4,5] is used to describe the composite particle emission in compound nucleus, which is included in the exciton model for the light composite particle emissions.

### 3 Theoretical calculations and analysis

The total cross sections for \( n^+{^{232}}\text{Th}, n^+{^{233}}, {^{235}}, {^{238}}\text{U}, n^+{^{237}}\text{Np}, n^+{^{239}}, {^{240}}, {^{242}}\text{Pu}, n^+{^{241}}, {^{243}}\text{Am}, n^+{^{249}}\text{Cm}, n^+{^{249}}\text{Bk} \) and \( n^+{^{249}}, {^{250}}, {^{251}}, {^{252}}\text{Cf} \) reactions are calculated using the global optical model potential parameters. The comparisons of calculated results with experimental data [6–32] for some nuclei are given in Fig. 5. These results show that the theoretical values are in good agreement with the experimental data, and the theoretical values of some nuclei are larger than the corresponding experimental data for incident neutron energies from 0.1 MeV to 1.0 MeV. The calculated results of total cross section for \( n^+{^{232}}\text{Th} \) and \( n^+{^{238}}\text{U} \) reactions are in good agreement with experimental data [6]. The total cross sections for other nuclei, where there are some experimental data for neutron energy above 1.0 MeV, are also calculated and analyzed, and similar results are obtained.

\[ \text{Fig. 5} \quad \text{Calculated total cross section (solid line) compared with experimental data [6–32] for } n^+{^{232}}\text{Th}, n^+{^{233}}, n^+{^{235}}, n^+{^{238}}\text{U}, n^+{^{237}}\text{Np}, n^+{^{239}}, n^+{^{240}}, n^+{^{242}}\text{Pu} \text{ and } n^+{^{241}}\text{Am} \text{ reactions. The results are offset by factors of } \times 2, \times 3, \ldots, \times 9 \]

The calculated results of nonelastic scattering cross sections for \( n^+{^{232}}\text{Th}, n^+{^{235}}, {^{238}}\text{U} \) and \( n^+{^{239}}\text{Pu} \) reactions are in good agreement with experimental data taken from EXFOR. The calculated results of elastic scattering cross sections for \( n^+{^{235}}, {^{238}}\text{U} \) reactions are in good agreement with experimental data taken from EXFOR for neutron energy below 20 MeV. The elastic scattering cross sections for \( n^+{^{232}}\text{Th}, {^{233}}\text{U} \) and \( {^{239}}\text{Pu} \) reactions are also calculated and compared with some existing experimental data for incident neutron energy
below 14.0 MeV. The calculation results fit well with the experimental data. There are no experimental data for other nuclei up to now as fas as the published literatures are concerned.

The calculated results of elastic scattering angular distribution for n+\(^{232}\)Th, \(^{233,235,238}\)U, \(^{237}\)Np and \(^{239}\)Pu reactions are in good agreement with the experimental data [35~37]. The experimental data of neutron elastic scattering angular distributions for natural U with incident neutron energy from 18 MeV to 120.0 MeV are given in Refs. [38,39]. The calculated results of elastic scattering angular distributions for n+\(^{233,235,238}\)U reactions are in good agreement with the experimental data.

Since the actinides are deformed nuclei and the energy of ground state rotational bands is small, it is almost impossible to separate neutron inelastic scattering data from elastic scattering data experimentally for actinide nuclei with incident energies above several MeV. The experimental data of neutron angular distribution including neutron elastic scattering and inelastic scattering of the first, second, third, and fourth excited states for different nuclei were individually given in different laboratories.

The inelastic scattering angular distributions for the first, second, third and fourth excited states of \(^{232}\)Th, \(^{233,235,238}\)U and \(^{239,242}\)Pu are calculated. The calculated results are in good agreement with experimental data. The inelastic scattering angular distributions for the first and second excited states of \(^{232}\)Th, \(^{235}\)U and \(^{239}\)Pu are also in good agreement with experimental data [36] for incident neutron energy of 3.4 MeV. The calculated results including elastic scattering angular distribution and inelastic scattering angular distribution of the first, second, third and fourth excited states for n+\(^{232}\)Th, \(^{235,238}\)U, and \(^{239}\)Pu reactions are compared with experimental data [35~60]. The calculated results are in good agreement with experimental data. Here the calculated results include elastic scattering angular distribution and inelastic scattering angular distribution of the first, second, third, and fourth excited states and the experimental data are taken from Ref. [60] for \(^{232}\)Th and \(^{235}\)U, and the comparisons are shown in Figs. 6 and 7.

The comparisons of calculated results of proton reaction cross sections with experimental data taken from Refs. [33,34] for \(^{232}\)Th and \(^{238}\)U are given in Fig. 8, in which good agreement can be seen. Since the experimental data taken from Refs. [35,36] are for p+U reactions, the calculated results of reaction cross sections for p+\(^{233,235,237}\)U reactions are also in reasonable agreement with experimental data. There is no experimental data for other nuclei up to now as far as the published literatures are concerned.

The calculated results of elastic scattering angular distributions are compared with experimental data [61~67] for p+\(^{232}\)Th and \(^{235,238}\)U reactions for incident proton energies from 13.0 MeV to 95.0 MeV. The calculated results fit experimental data very well for all
XU Yongli et al.: The Nucleon-Actinide Global Optical Model Potential Below 300 MeV

energy points. Fig. 9 gives the calculated results for $p^{+232}$Th reactions only.

The calculated results of inelastic scattering angular distributions of the first, second, third and fourth excited states are compared with experimental data \[^{61\sim63}\] for $p^{+232}$Th reactions for incident proton energies from 20.0 MeV to 65.0 MeV. The calculated results fit experimental data well for all excited states and incident proton energy points. The inelastic scattering angular distributions of $p^{+238}$U reactions are also calculated, and similar results are obtained. Fig. 10 gives only the calculated results of the first excited states for $p^{+232}$Th reactions.

The obtained optical model potential is applied to $p^{+232}$Th reaction. All reaction cross sections, energy spectra and double differential cross sections of neutron, proton, deuteron, triton, and alpha-particle emission are calculated, and theoretical calculations are compared with existing experimental data.

The comparison of calculated results of $(p, n)$ reaction cross sections with experimental data are given in Fig. 11, and good agreement is found in the entire energy region. The calculated results of $(p, \alpha)$ reaction cross sections pass the error of the experimental data \[^{69}\]. The cross sections of $(p, d)$ and $(p, t)$ reactions are less than 20.0 mb, and there are no experimental data. The calculated results of $(p, 2n)$ reaction cross sections are compared with experimental data \[^{70}\] as shown in Fig. 12, and good agreement is also found. The calculated results for $(p, 3n)$ and $(p, 6n)$ reaction cross sections are in good agreement with experimental data \[^{71\sim73}\].

The experimental data of fission cross sections were given in different laboratories, and there are significant differences among them. The calculated results are in good agreement with the experimental data taken from Refs. \[^{74\sim76}\], as shown in Fig. 13. They are similar to the results of systematics taken from Ref. \[^{77}\].

\[\text{Fig. 9} \quad \text{Calculated elastic scattering angular distributions (solid line) compared with experimental data \[^{61\sim63}\] for } p^{+232}\text{Th reactions. The results are offset by factors of } \times10, \times10^2, \text{and } \times10^3\]

\[\text{Fig. 10} \quad \text{Calculated inelastic scattering angular distributions of the first excited state (solid line) compared with experimental data \[^{61\sim64,68}\] for } p^{+232}\text{Th reactions. The results are offset by factors of } \times10, \times10^2, \text{and } \times10^3\]

\[\text{Fig. 11} \quad \text{Calculated } (p, n) \text{ reaction cross sections (solid line) compared with experimental data}\]

\[\text{Fig. 12} \quad \text{Calculated } (n, 2n) \text{ reaction cross sections (solid line) compared with experimental data}\]

The experimental data of fission cross sections were given in different laboratories, and there are significant differences among them. The calculated results are in good agreement with the experimental data taken from Refs. \[^{74\sim76}\], as shown in Fig. 13. They are similar to the results of systematics taken from Ref. \[^{77}\].
The calculated results for neutron emission spectra at incident proton energy of 62.9 MeV are compared and found in good agreement with the experimental data\cite{78}, as shown in Fig. 14. The contribution of fission reaction, the preequilibrium and equilibrium emission process are also shown in Fig. 14. The shape and magnitude of the calculated curve of alpha emission spectra at incident proton energy of 70.7 MeV are in good agreement with those of experimental data\cite{79}.

The experimental data\cite{79,80} of the double differential cross sections of triton emission at triton emission energy 55.0 MeV were given for incident proton energy of 70.7 MeV. Calculated results and experimental data are shown in Fig. 15 for different emission angles. The calculated results are in good agreement with experimental data. There are no experimental data of proton-induced energy spectra and double differential cross sections of proton and deuteron emission up to now.

4 Conclusions

We have presented a set of nucleon global optical model potential parameters for the actinide region (the charge number range is 89≤Z≤100 and the mass range is 220≤A≤260) for nucleon energies up to 300 MeV by using the experimental data of $^{232}$Th and $^{238}$U. The obtained model potential parameters are applied to the p+$^{232}$Th reaction. All cross sections, the energy spectra and double differential cross sections are calculated, and are found in good agreement with available experimental data. The optical model potential developed here may find direct applications to theoretical nuclear model calculations and experiment analyses.

References
